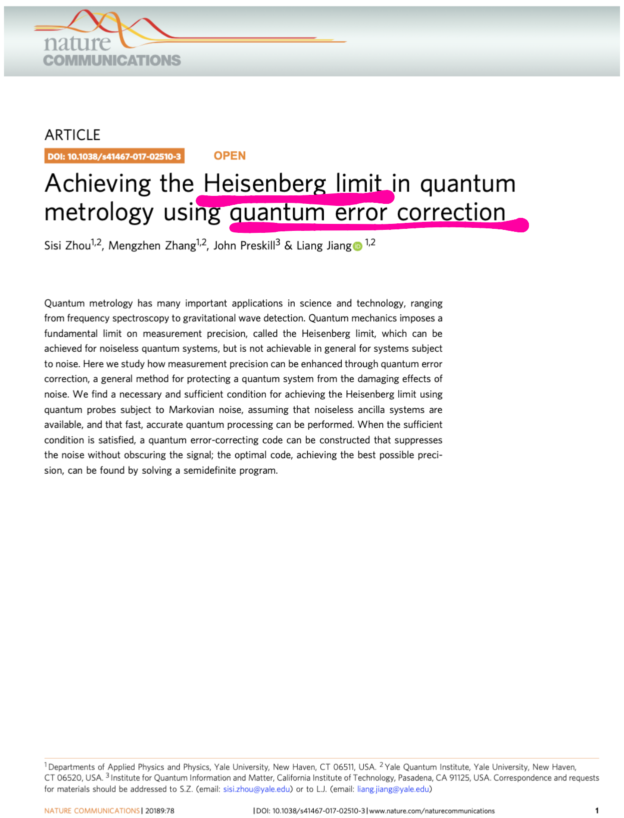
From quantum metrology to quantum computation and back

Robert Raussendorf,
Leibniz Universität Hannover

241. Heraeus Seminar

Three excursions

Quantum sensing and computation
— Why together?



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ARTICLE

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Achieving the Heisenberg limit in quantum metrology using quantum error correction

Sisi Zhou[1,2], Mengzhen Zhang[1,2], John Preskill[3] & Liang Jiang[1,2]

Quantum metrology has many important applications in science and technology, ranging from frequency spectroscopy to gravitational wave detection. Quantum mechanics imposes a fundamental limit on measurement precision, called the Heisenberg limit, which can be achieved for noiseless quantum systems, but is not achievable in general for systems subject to noise. Here we study how measurement precision can be enhanced through quantum error correction, a general method for protecting a quantum system from the damaging effects of noise. We find a necessary and sufficient condition for achieving the Heisenberg limit using quantum probes subject to Markovian noise, assuming that noiseless ancilla systems are available, and that fast, accurate quantum processing can be performed. When the sufficient condition is satisfied, a quantum error-correcting code can be constructed that suppresses the noise without obscuring the signal; the optimal code, achieving the best possible precision, can be found by solving a semidefinite program.

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Manny Knill works at NIST!

Hard fact / Soft fact

Excursion #1

•

Heisenberg vs. Grover

Heisenberg and standard quantum limit

Setting: Magnetic field of unknown strength B—measure it!

The Hamiltonian is

$$H = -B \sum_{i=1}^{n} \sigma_z^{(i)}.$$

Couple magnetic field B to spin(s).

Option 1

$$H = -B \sum_{i=1}^{n=1} \sigma_z^{(i)}.$$

- Choose $n = 1$ (one single spin only).
- Prepare initial state $(|0\rangle + |1\rangle)/\sqrt{2}$, evolve H for some fixed time t, measure observable σ_x.
- Repeat N times.

Accuracy:

$$\boxed{\Delta B \propto \frac{1}{\sqrt{N}}}$$

This is the standard quantum limit

Option 2

$$H = -B \sum_{i=1}^{N} \sigma_z^{(i)}.$$

- Choose $n = N$ (prepare N spins).
- Prepare initial state $(|00..0\rangle + |111..1\rangle)/\sqrt{2}$, evolve H for some fixed time t, measure observable $\overline{X} := \bigotimes_{i=1}^{N} \sigma_x^{(i)}$.
- Repeat once.

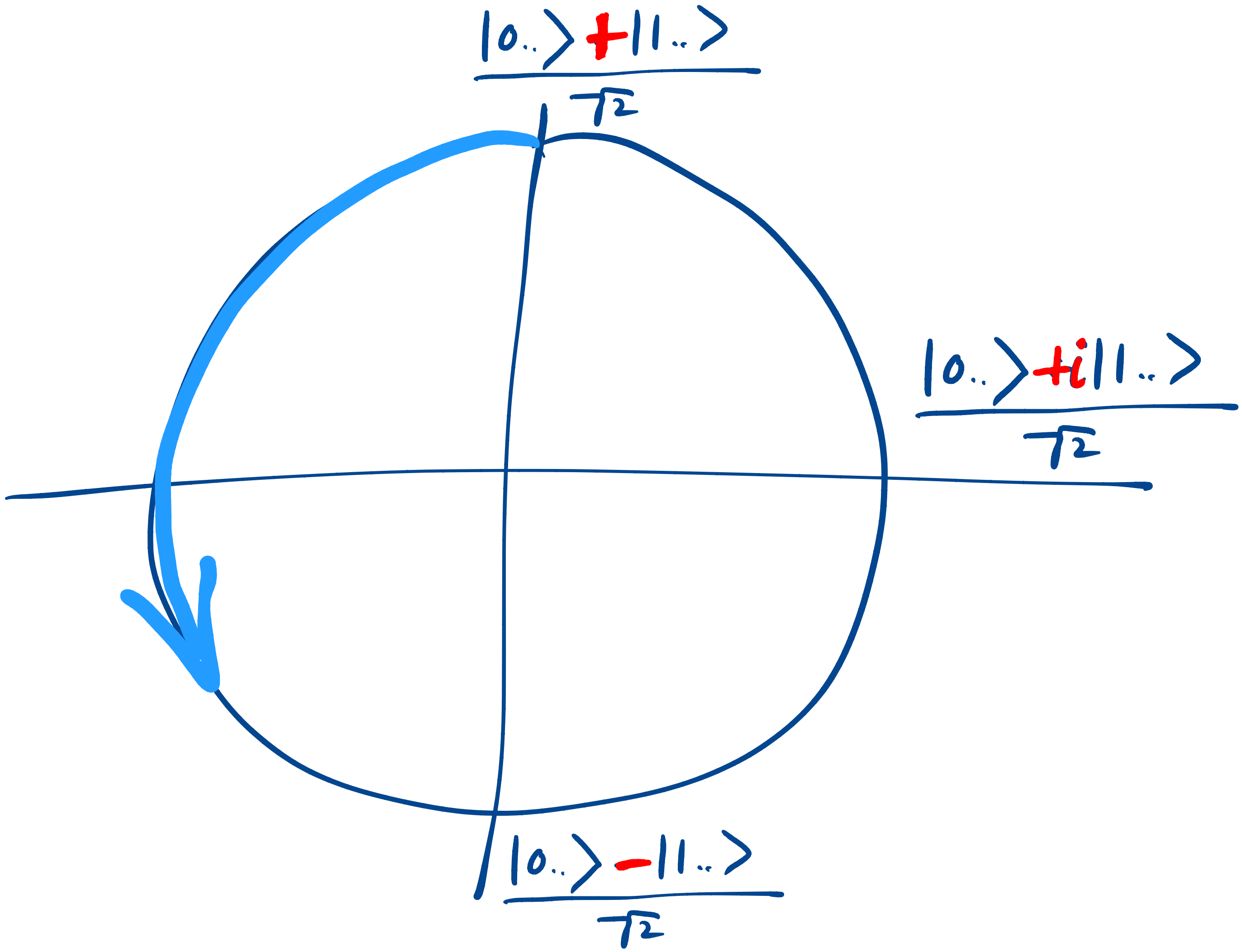
Accuracy:

$$\boxed{\Delta B \propto \frac{1}{N}}$$

This is the Heisenberg limit. It provides a quadratic speedup.

In both cases ..

.. evolution happens in a 2d Hilbert space.



Grover's data base search

Who's number is 0162 7615421?

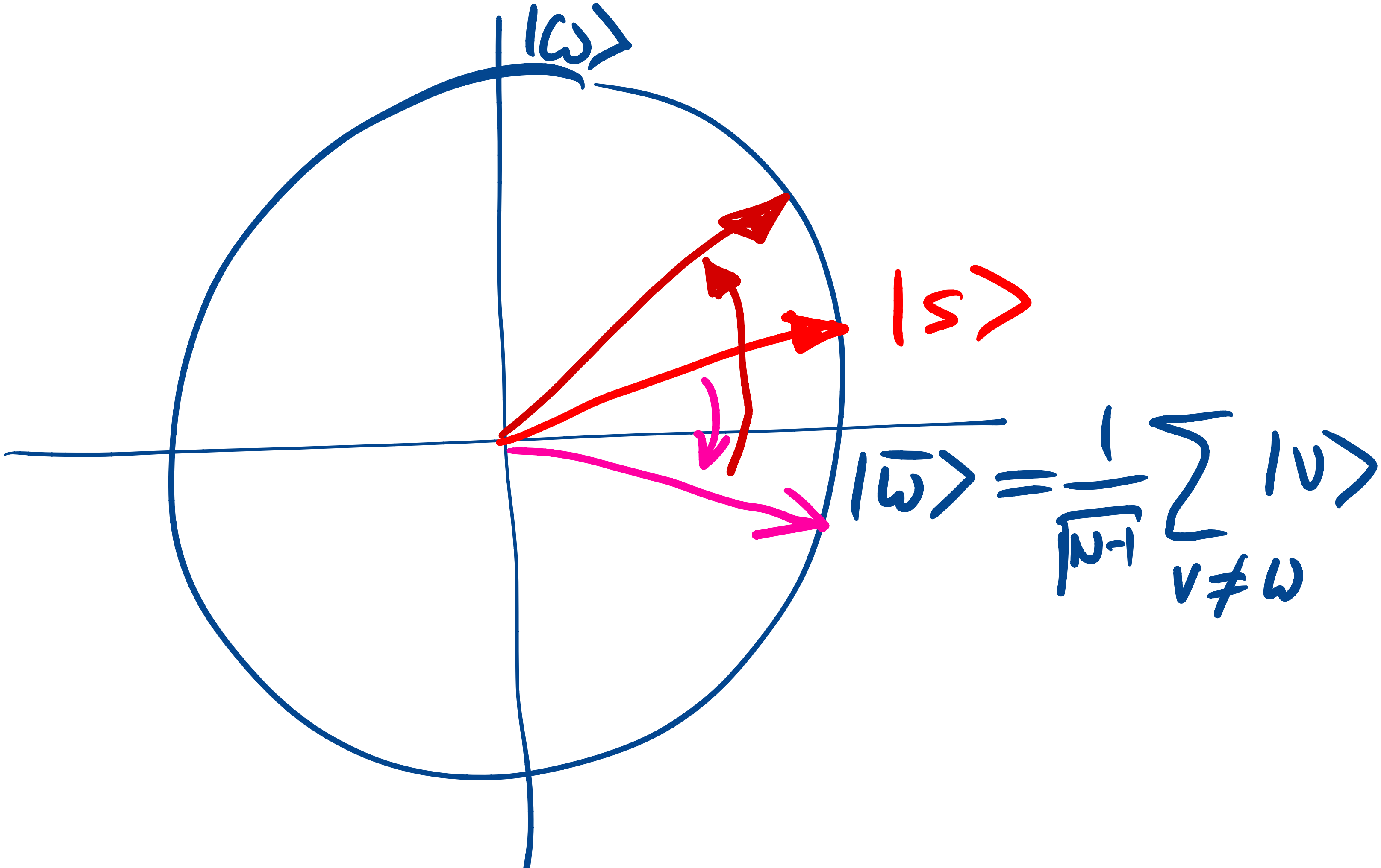
Find a data base entry w among N, using a quantum oracle

$$\begin{aligned} U_{\text{oracle}}|w\rangle &= -|w\rangle, && \textit{target} \\ U_{\text{oracle}}|v\rangle &= |v\rangle, && \forall v \neq w. \end{aligned}$$

- Grover's algorithm does this in $\propto \sqrt{N}$ oracle calls.
- Classically, require $\propto N$ steps.

Again, a quadratic speedup. Is the analogy superficial, or does it have a basis?

Grover too: evolution in 2d



We observe: *Coherent accumulation of phase powers both Heisenberg and Grover*

Lessons from Excursion #1

Heisenberg

Grover

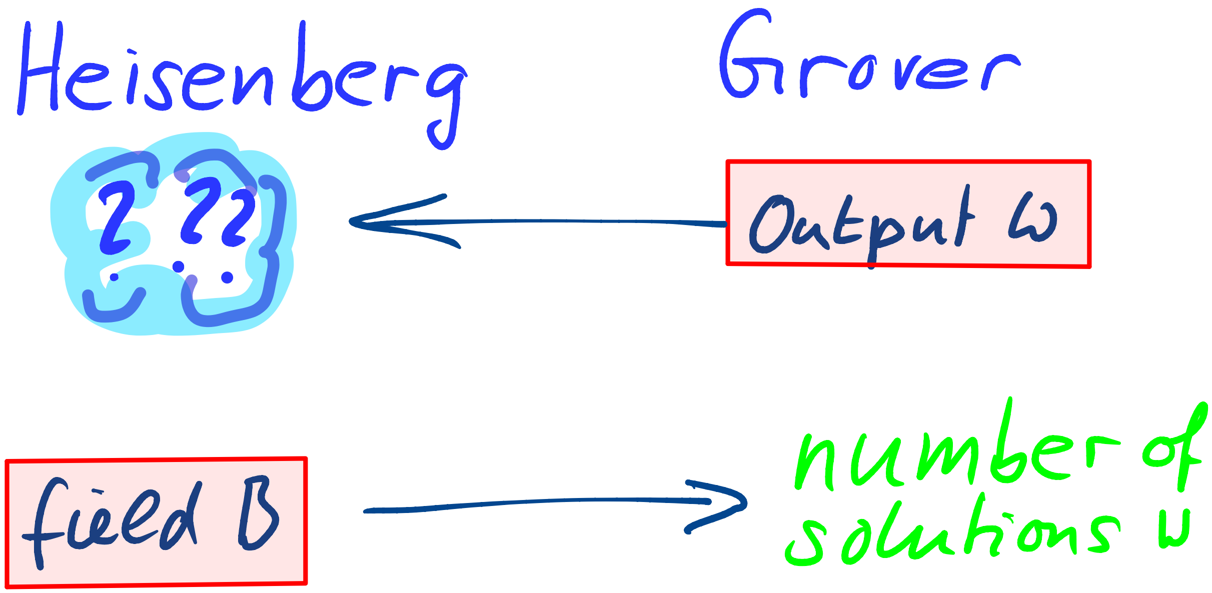
$\boxed{\text{Output } \omega}$

$\boxed{\text{field } B}$

Lessons from Excursion #1

Heisenberg		Grover
{???}	$\longleftarrow$	$\boxed{\text{Output } \omega}$
$\boxed{\text{field } B}$	$\longrightarrow$	number of solutions ω

Lessons from Excursion #1



- Want w to arise with probability $p(w) > 1/2$, say. However, you do not care about the precise value of $p(w)$.
- You (the Grover-operator) are interested in sampling from p, not in knowing p.

Excursion #2

•

The threshold theorem of fault-tolerant quantum computation

Threshold Theorem

Theorem: If the error of every operation in a quantum computation is below a critical constant value ϵ, then arbitrarily accurate logical gates and measurements can be performed, and arbitrarily long quantum computation is possible and efficient.

Broadly accepted version is of 2005 (Aliferis & Preskill)

Threshold Theorem

looks like budget stuff

Looks like precision!

Theorem: If the error of every operation in a quantum computation is below a critical constant value ϵ, then arbitrarily accurate logical gates and measurements can be performed, and arbitrarily long quantum computation is possible and efficient.

Threshold Theorem

looks like budget stuff

looks like precision!

Theorem: If the error of every operation in a quantum computation is below a critical constant value ϵ, then arbitrarily accurate logical gates and measurements can be performed, and arbitrarily long quantum computation is possible and efficient.

bad quality in → good quality out

(lots of)

10^{-4} is good enough

better

— 10^{-2}: Threshold

— 10^{-4}: Safe place

— 10^{-6}: No need to be here

Accuracy of what?

.. of transforming the quantum state

- The procedure of QEC does not usually involve obtaining expectation values of observables

No corresponding accuracy needed.

only the outcomes of individual measurement events count

The two rules of quantum measurement

1882 — 1970

$$p(s) = \mathrm{Tr}(\Pi_s \rho)$$

Max Born

1902 — 1984

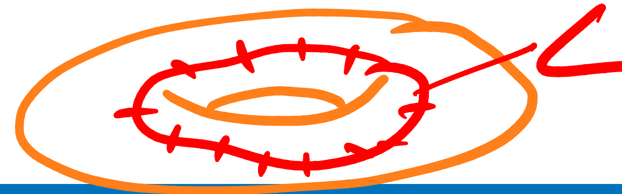
$$|\psi\rangle \longmapsto \frac{\Pi_s |\psi\rangle}{\sqrt{\langle\psi|\Pi_s|\psi\rangle}}$$

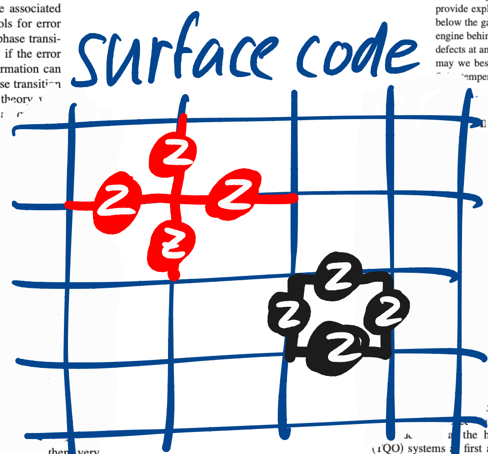
P.A.M. Dirac

Does metrology live mostly here?

Quantum computing lives mostly here!

Of battles past





JOURNAL OF MATHEMATICAL PHYSICS VOLUME 43, NUMBER 9 SEPTEMBER 2002

Topological quantum memory[a)]

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(Received 25 October 2001; accepted for publication 16 May 2002)

We analyze *surface codes*, the topological quantum error-correcting codes introduced by Kitaev. In these codes, qubits are arranged in a two-dimensional array on a surface of nontrivial topology, and encoded quantum operations are associated with nontrivial homology cycles of the surface. We formulate protocols for error recovery, and study the efficacy of these protocols. An order-disorder phase transition occurs in this system at a nonzero critical value of the error rate; if the error rate is below the critical value (the *accuracy threshold*), encoded information can be protected arbitrarily well in the limit of a large code block. ...

I. INTRODUCTION

PHYSICAL REVIEW B **77**, 064302 (2008)

Autocorrelations and thermal fragility of anyonic loops in topologically quantum ordered systems

Zohar Nussinov[1] and Gerardo Ortiz[2]
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(Received 24 September 2007; revised manuscript received 27 December 2007; published 14 February 2008)

Are systems that display topological quantum order (TQO), and have a gap to excitations, hardware fault-tolerant at finite temperatures? We show that in models that display low d-dimensional gaugelike symmetries, such as Kitaev's and its generalizations, the expectation value of topological symmetry operators vanishes at *any* nonzero temperature, a phenomenon that we coined *thermal fragility*. The autocorrelation time for the nonlocal topological quantities in these systems may remain finite even in the thermodynamic limit. We provide explicit expressions for the autocorrelation functions in Kitaev's toric code model. If temperatures far below the gap may be achieved then these autocorrelation times, albeit finite, can be made large. The physical engine behind the loss of correlations at large spatial and/or temporal distance is the proliferation of topological defects at any finite temperature as a result of a *dimensional reduction*. This raises an important question: How may we best quantify the degree of protection of quantum information in a topologically ordered system at ... temperature?

II. LANDAU ORDERS VS TQO

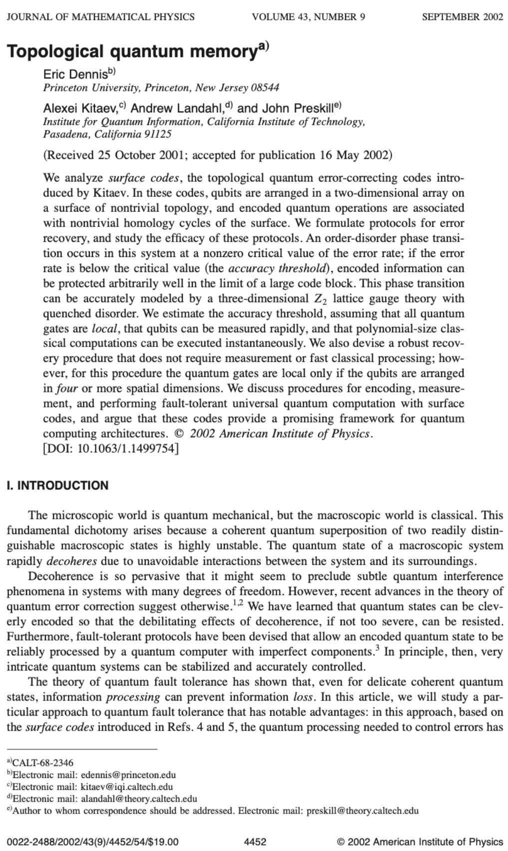
$\boxed{T_{mem} \propto \exp(cL)}$

$\boxed{T_{mem} = \mathrm{const}(L)}$

[L] E. Dennis, A. Landahl, A. Kitaev, J. Preskill, J. Math Phys 43, 4452 (2002).

[R] Z. Nussinov and G. Ortiz, Phys Rev B 77, 064302 (2008).

Dennis et al.



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[DOI: 10.1063/1.1499754]

I. INTRODUCTION

The microscopic world is quantum mechanical, but the macroscopic world is classical. This fundamental dichotomy arises because a coherent quantum superposition of two readily distinguishable macroscopic states is highly unstable. The quantum state of a macroscopic system rapidly *decoheres* due to unavoidable interactions between the system and its surroundings.

Decoherence is so pervasive that it might seem to preclude subtle quantum interference phenomena in systems with many degrees of freedom. However, recent advances in the theory of quantum error correction suggest otherwise.[1,2] We have learned that quantum states can be cleverly encoded so that the debilitating effects of decoherence, if not too severe, can be resisted. Furthermore, fault-tolerant protocols have been devised that allow an encoded quantum state to be reliably processed by a quantum computer with imperfect components.[3] In principle, then, very intricate quantum systems can be stabilized and accurately controlled.

The theory of quantum fault tolerance has shown that, even for delicate coherent quantum states, information *processing* can prevent information *loss*. In this article, we will study a particular approach to quantum fault tolerance that has notable advantages: in this approach, based on the *surface codes* introduced in Refs. 4 and 5, the quantum processing needed to control errors has

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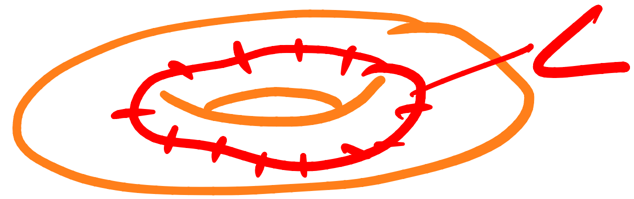

* Active error correction by stabilizer mmnt.

* Memory time $T_{mem} \propto \exp(cL)$

Dennis, A. Landahl, A. Kitaev, J. Preskill, J. Math Phys 43, 4452 (2002).

Nussinov and Ortiz


PHYSICAL REVIEW B **77**, 064302 (2008)

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DOI: 10.1103/PhysRevB.77.064302 PACS number(s): 05.30.−d, 03.67.Pp, 05.30.Pr, 11.15.−q

I. INTRODUCTION

The perseverance of information over long times in the simplest of memory devices is related to the existence of large associated autocorrelation times. The storage of information is intimately tied to the breaking of ergodicity at scales much smaller than the autocorrelation time. Classical information can be reliably stored in magnetically or in electrically (permanently) polarized materials. From the physicist's perspective, this reliability may be directly linked to the existence of an *order parameter* (its *macroscopic* magnetization or polarization) which characterizes a *collective* and robust property of the material below an ordering transition temperature. At its core, nonergodicity implies the existence of a *generalized order parameter* (e.g., the overlap parameter of spin glasses).

The reliable storage of quantum information is a real challenge. The uncontrolled interactions between a quantum system and its environment or measurement apparatus introduce noise (errors) in the system leading to decoherence of pure quantum superposition states. Fortunately, quantum states can, in principle, be encoded fault tolerantly and be *protected* against decoherence, thus preventing loss of information.[1] This idea lies at the heart of topological quantum order (TQO) systems as first advanced by Kitaev.[2] Assuming that errors are of a *local* nature, topological quantum memories (e.g., surface codes[2]) seem to be intrinsically stable because of *physical* fault tolerance to weak quasilocal perturbations. However, are these quantum memories robust to thermal effects?

In this work, we analyze the effect of temperature on zero-temperature ($T=0$) topologically ordered quantum systems,[3,4] such as Kitaev's toric code[2] and honeycomb models[5] and generalizations thereof. To this end, we need to present two concepts that were introduced in our previous work.[6] One is the concept of finite-T TQO, and the other of rank-n TQO. In that same work we studied the thermal fragility of topological operators in $D=2$ lattice models. Our results[6] concerning the singular character of the $T=0$ TQO in one notable system (Kitaev's toric code model) have later been reaffirmed in work by Castelnovo and Chamon[7] in their study of the topological entanglement entropy. In the present work we will present extensions of our ideas to higher spatial dimensions D and expand on the physical reasons leading to thermal fragility. In particular, we show that a general $\mathbb{Z}_k$ gauge theory in D spatial dimensions in a system with periodic boundary conditions displays rank-$n=k^D$ TQO. Nevertheless, although a thermodynamic phase transition may occur, the system is thermally fragile. We investigate not only the thermodynamic but also the dynamical aspects of thermal fragility, and in cases such as Kitaev's toric code model we also obtain exact analytic time-dependent results thanks to our duality mappings.[6]

II. LANDAU ORDERS VS TQO

Before defining TQO, and to put this latter concept in perspective, let us briefly review the rudiments of a Landau order parameter. The Landau order parameter, a macroscopic property measuring the degree of order in a state of matter, is customarily associated with the breaking of a global symmetry. Thus the existence of an order parameter is directly attached to the phenomenon of spontaneous symmetry breaking (SSB). This concept, that involves an infinite number of degrees of freedom, is so fundamental to condensed matter and particle physics that many excellent textbooks (see, for example, Ref. 8) have spent entire chapters (or even a full book[9]) describing it. For the present purposes, we illustrate the concept in the simple case of a ferromagnet. A piece of iron at high temperatures it is in a *disordered* paramagnetic phase. Below a certain temperature T_c the system *orders*, i.e., it magnetizes, and with the appearance of the order parameter (magnetization) there is a breaking of the rotational symmetry.[10] In the (ferro)magnetic phase there is a net magnetization $\mathbf{M}$ that persists all the way to zero temperature (where it attains its maximal value). The magnetization can,

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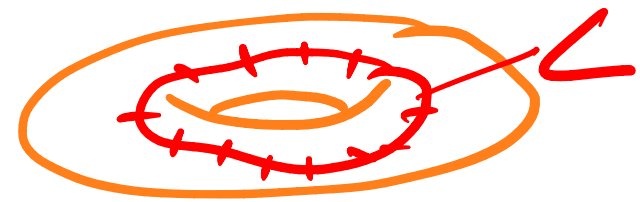
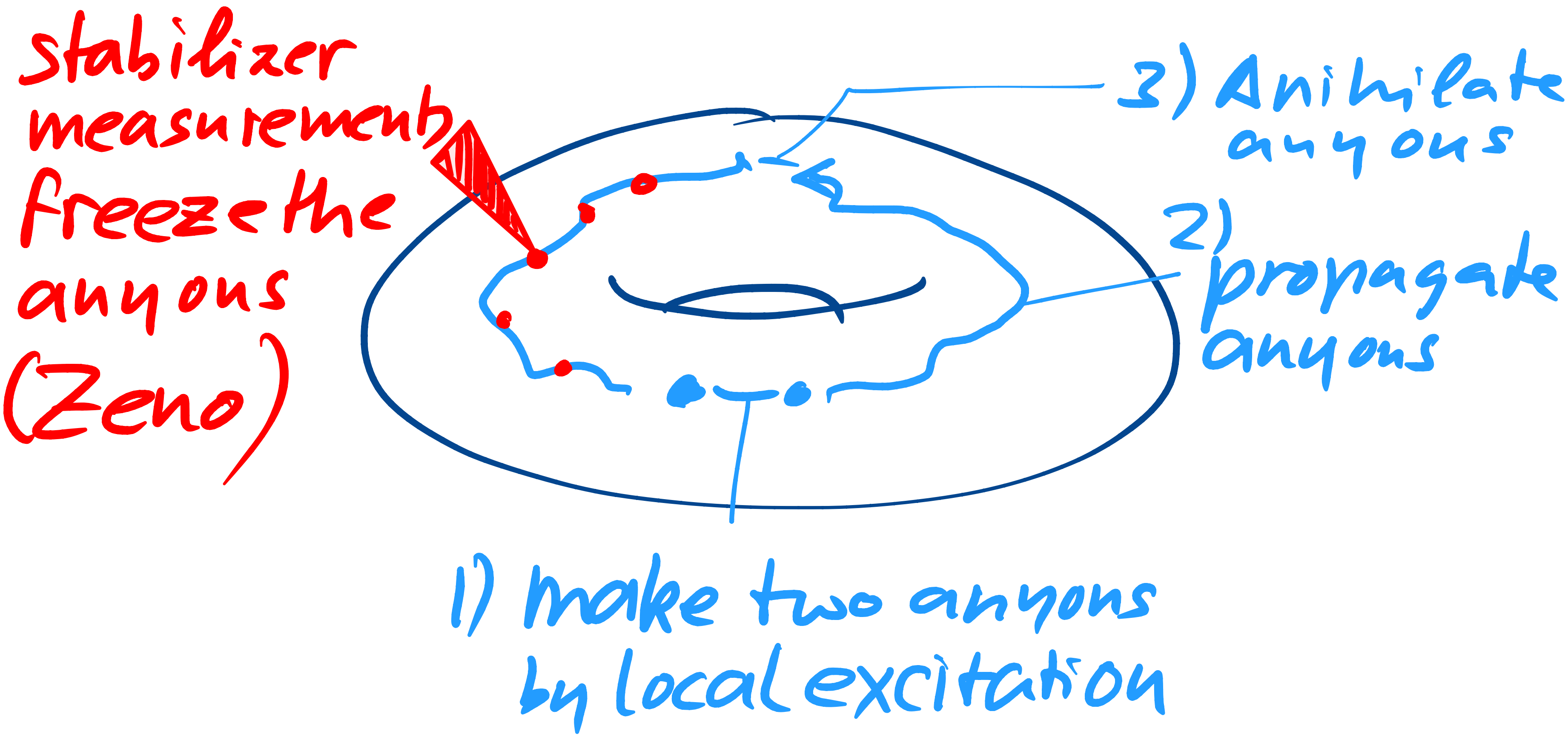


* Passive error protection by Hamiltonian with finite gap.

* $T_{mem} = const(L)$

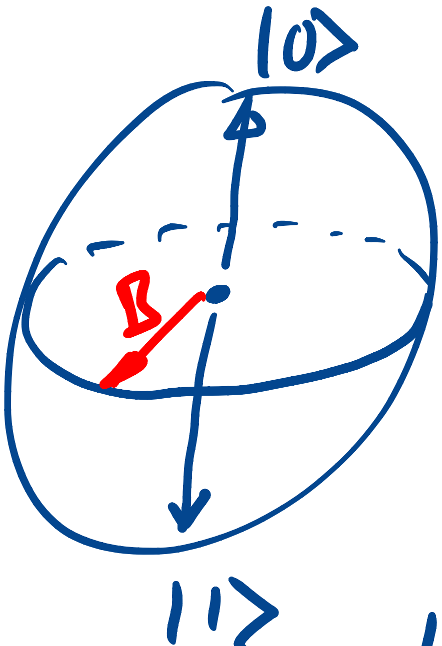
[R] Z. Nussinov and G. Ortiz, Phys Rev B 77, 064302 (2008).

Intuition about this



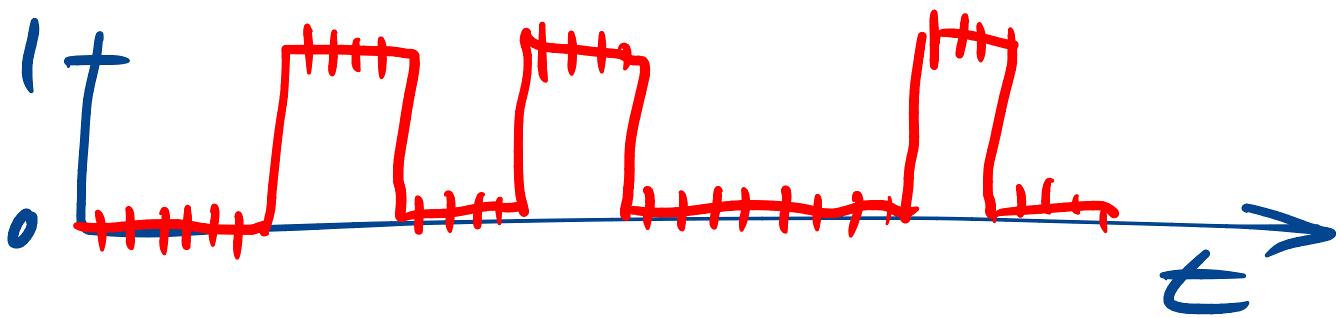
Both are correct, consider different settings.

About the Zeno effect



- $H = B\sigma_x$
- frequent measurement in basis $\{|0\rangle, |1\rangle\}$.

Zeno:



Zeno & flip:

QEC as Zeno effect

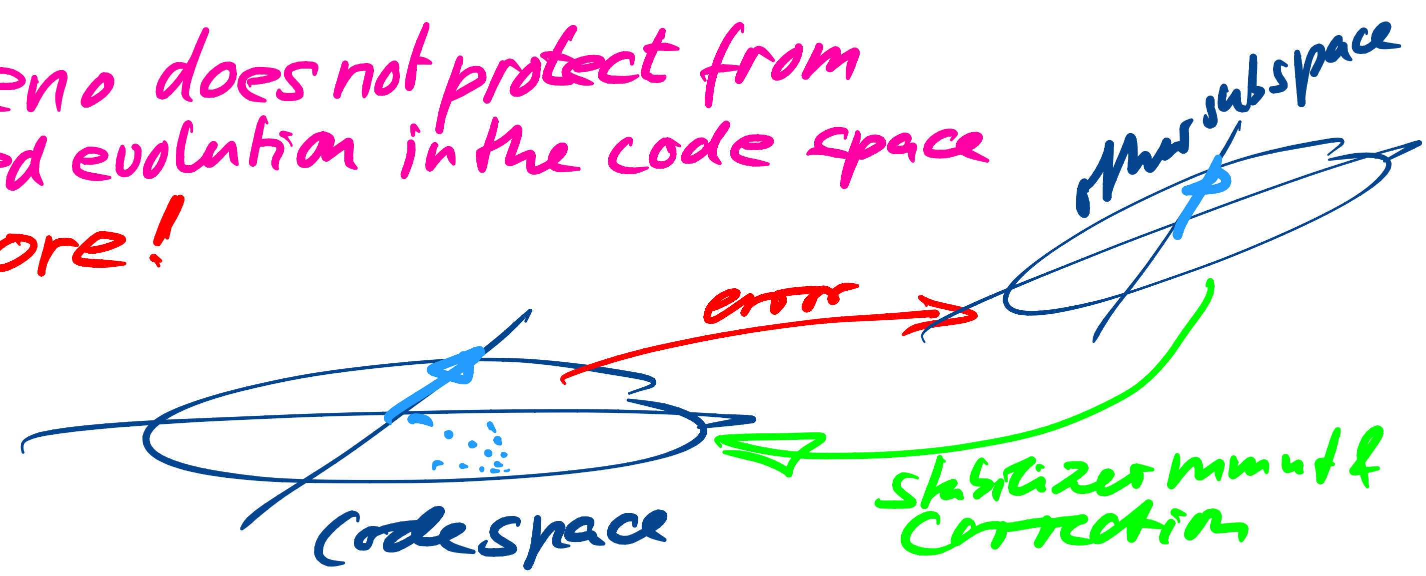
* QEC is like the "corrected" Zeno effect, but with one difference:

- The measurements of QEC are degenerate.

- QEC does not differentiate inside the code space. (On purpose)

→ QEC-Zeno does not protect from undesired evolution in the code space

need more!



QEC as Zeno effect

↳ QEC-Zeno does not protect from undesired evolution in the code space
need more!

Need a special relation between noise and code. Described by the Knill-Laflamme conditions:

errors   states in code space

$$\langle i | E_a^\dagger E_b | j \rangle = \langle i | j \rangle \cdot \sigma(E_a, E_b)$$

complex number

If satisfied, then QEC-Zeno protects from evolution in the code space.

↳ Protected memory

Summary of Excursion 2

* QEC is a Zeno-type thing

* You measure to project, not to learn.
  - Forbidden learning: You MUST NOT learn the encoded state.
  - Pointless learning: You could learn the error model, but you do not care very much. (QEC is not fine-tuned)

Excursion #3

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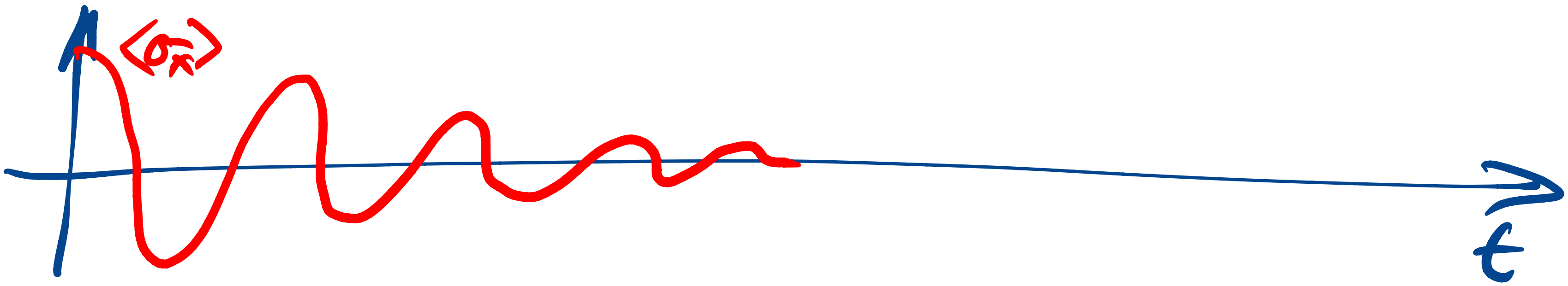
Error-corrected sensing

The problems (2)



* Decoherence kills the signal

$\Longrightarrow$ back to standard quantum limit

* QEC kills decoherence

... but the signal along with it

so not a general solution

Error-corrected sensing

Setting:

signal   noise

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{k=1}^{r}\left(L_k \rho L_k^{\dagger} - \frac{1}{2}\{L_k^{\dagger}L_k, \rho\}\right)$$

Hamiltonian-not-in-Lindblad span (HNLS) condition:

$$H \notin \mathrm{Span}(I, L_k, L_k^{\dagger}, L_k^{\dagger}L_j)$$

Theorem*: The Heisenberg limit is achievable if and only if the HNLS condition holds.

[*]: S. Zhou, M. Zhang, J. Preskill, L. Jiang, Nature Commun.

Back to QC: The Eastin-Knill Theorem

The HNLS condition has a counterpart in FTQC:

Theorem: (Eastin-Knill) No quantum code can unitarily and transversally implement a universal gate set.

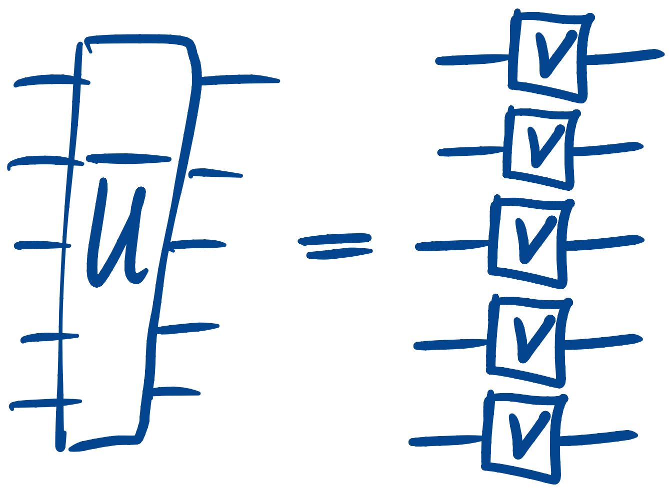
Is this a killer for quantum fault-tolerance?

no: mind the assumptions!

B. Eastin and E. Knill, Phys. Rev. Lett. (2009)

Back to QC: The Eastin-Knill Theorem

Transversal & unitary encoded gates:



encoded gate action happens locally on individual qubits.

Gist of Proof: Suppose you were universal

- Can find U such that $V \approx I$
- But that's an error. ↯

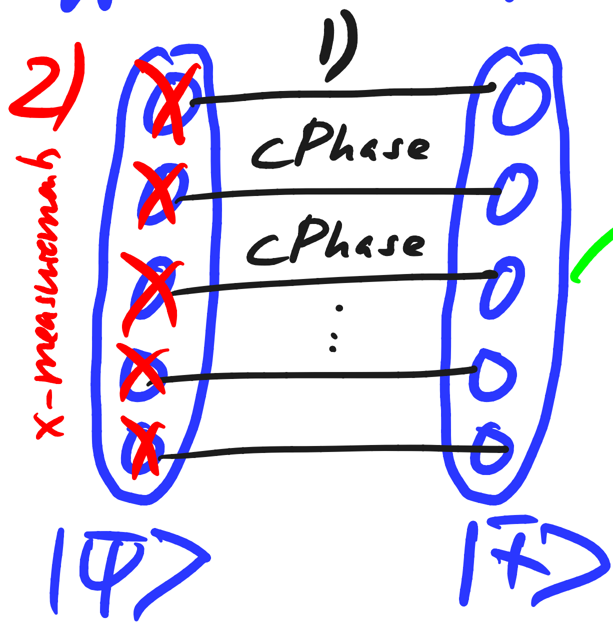
Like ¬HNLS: You correct the signal with the noise.

Measurement overcomes Eastin-Knill

Dispense the assumption of "unitary".

Measurement helps you do a universal set.

Hadamard for all CSS codes:

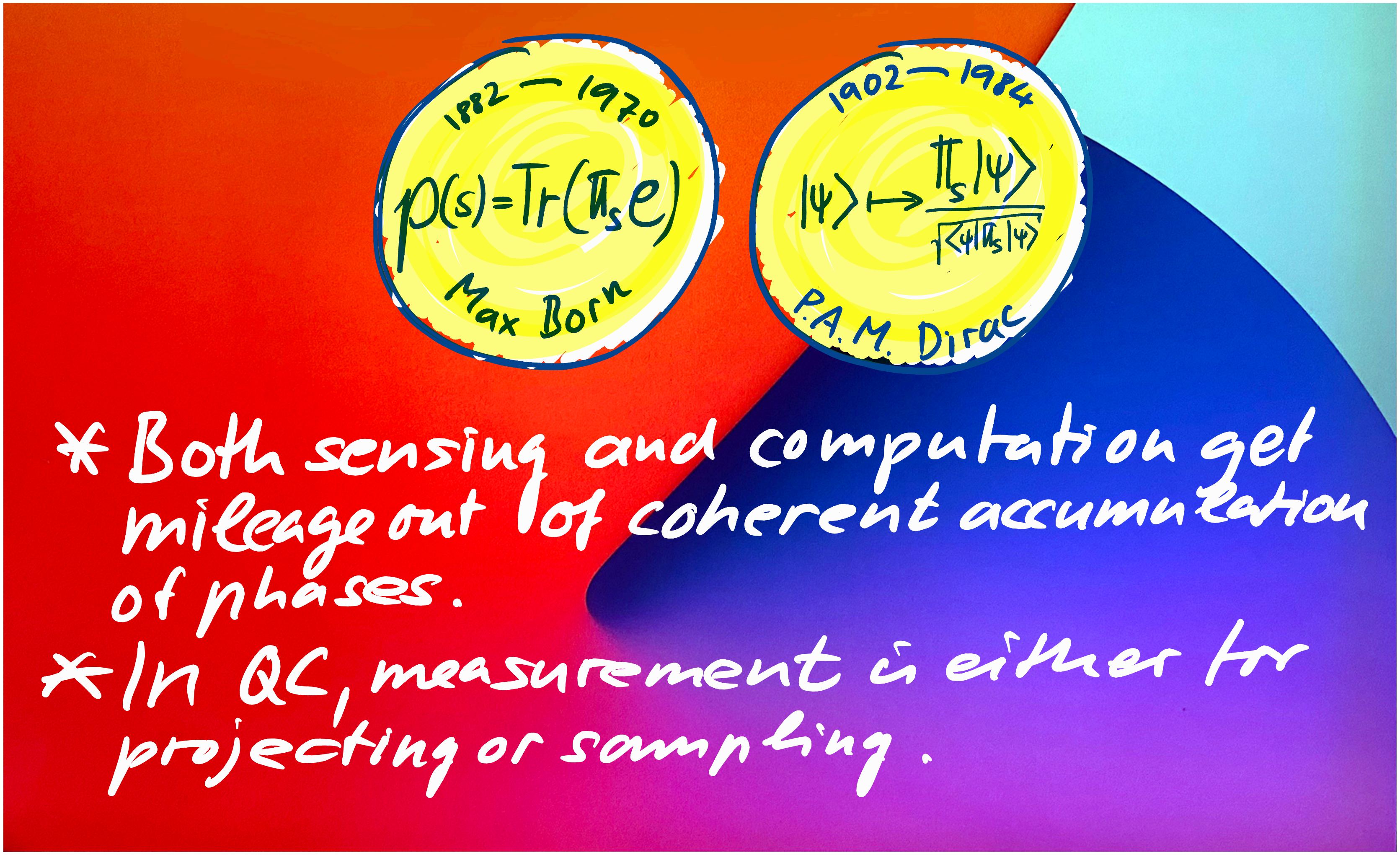


$H|\Psi\rangle$

15-qubit CSS code $RM_*(4,1)$ has transversal T-gate

Universal

To sum up ..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$|\psi\rangle \mapsto \frac{\Pi_s|\psi\rangle}{\sqrt{\langle\psi|\Pi_s|\psi\rangle}}$
P.A.M. Dirac
* Both sensing and computation get mileage out of coherent accumulation of phases.
* In QC, measurement is either for projecting or sampling.