The Precision Limits of a Quantum Momentum Measurement and its Influence on Quantum Technologies

Theory and Experiment

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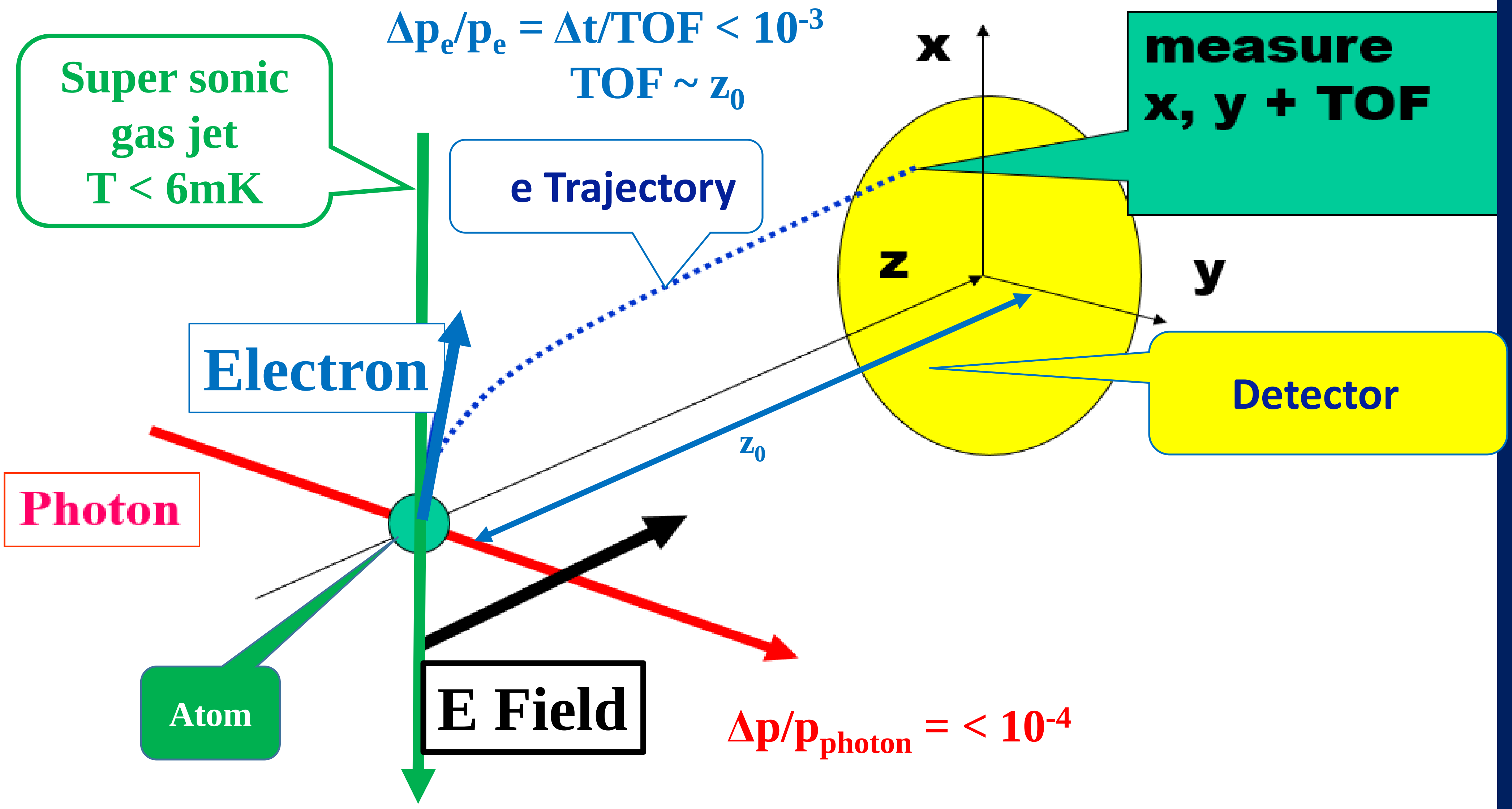
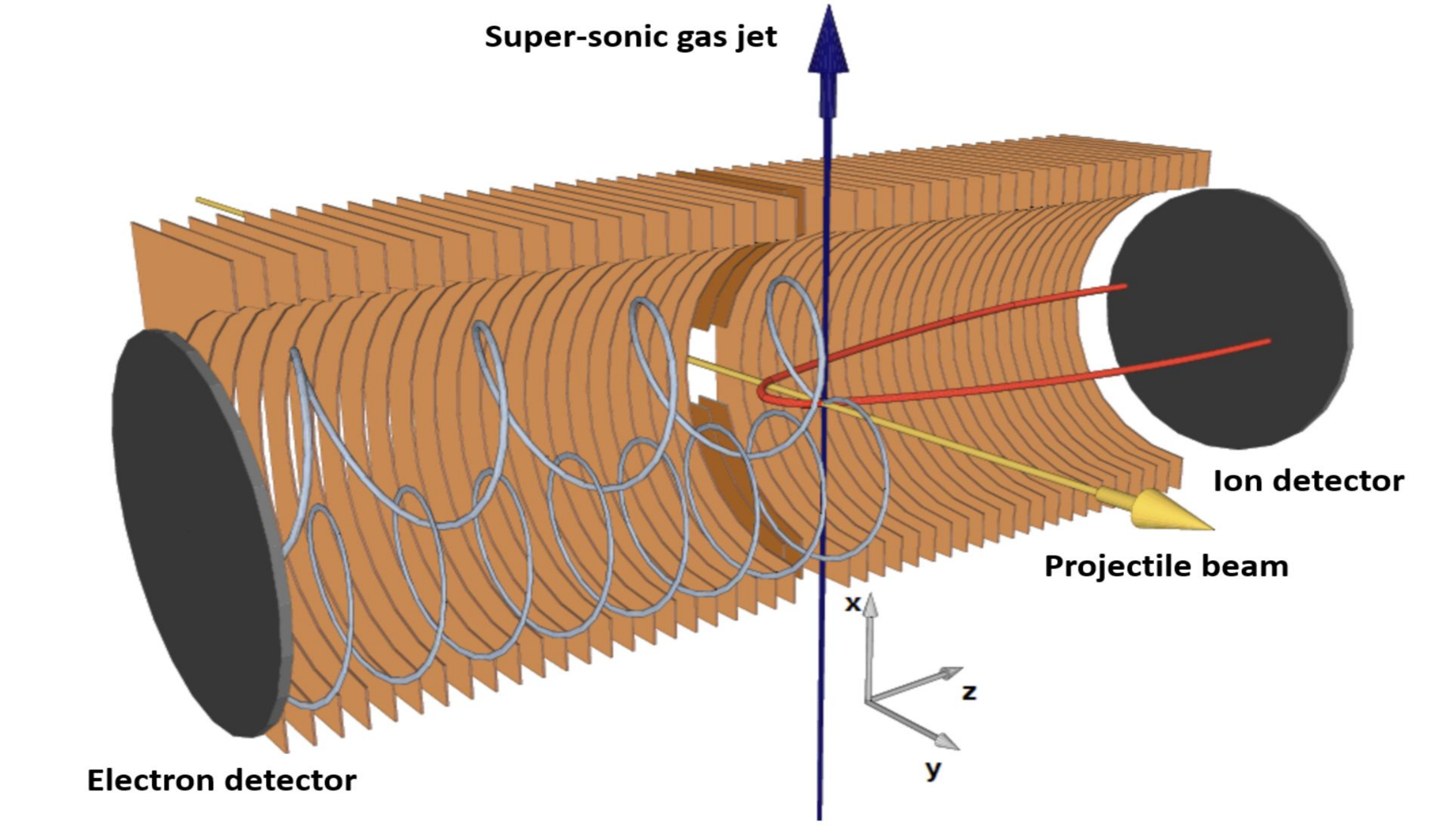
Momentum measurement by Time-of-Flight TOF




Super-sonic gas jet
Ion detector
Projectile beam
Electron detector
x
z
y

Basic rules for a successful Quantum measurement

The basis of a modern experimental investigation with state-of-the-art multi-coincidence detection technique is a single-event measurement!

Each experimental investigation is the sum of numerous single-event measurements, where in each single event all particles are dynamically entangled due to time dependent conservation laws!

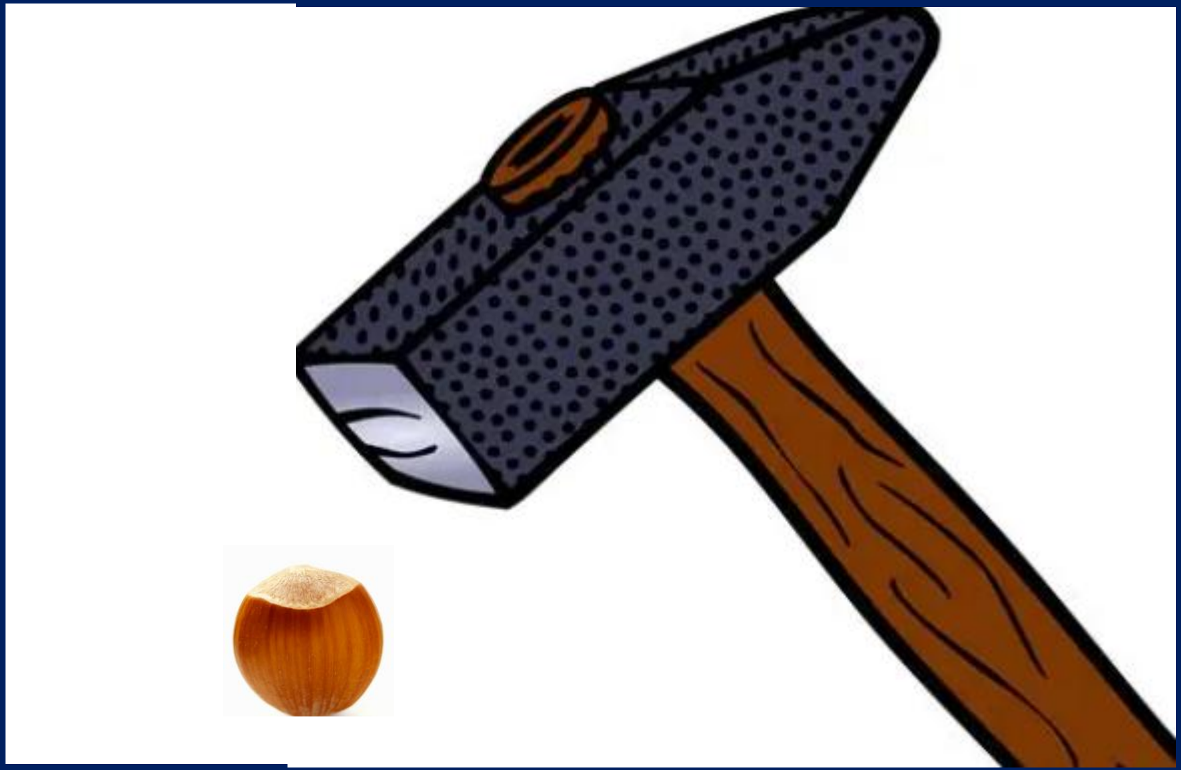
The sum of all these events is, however, a statistical average and all entanglement has disappeared.

The measureable parameters in a single event are:
Momentum (the complete vector),
angular momentum (Quantum numbers),
energy?
position ??????,
…

To obtain good or excellent resolution

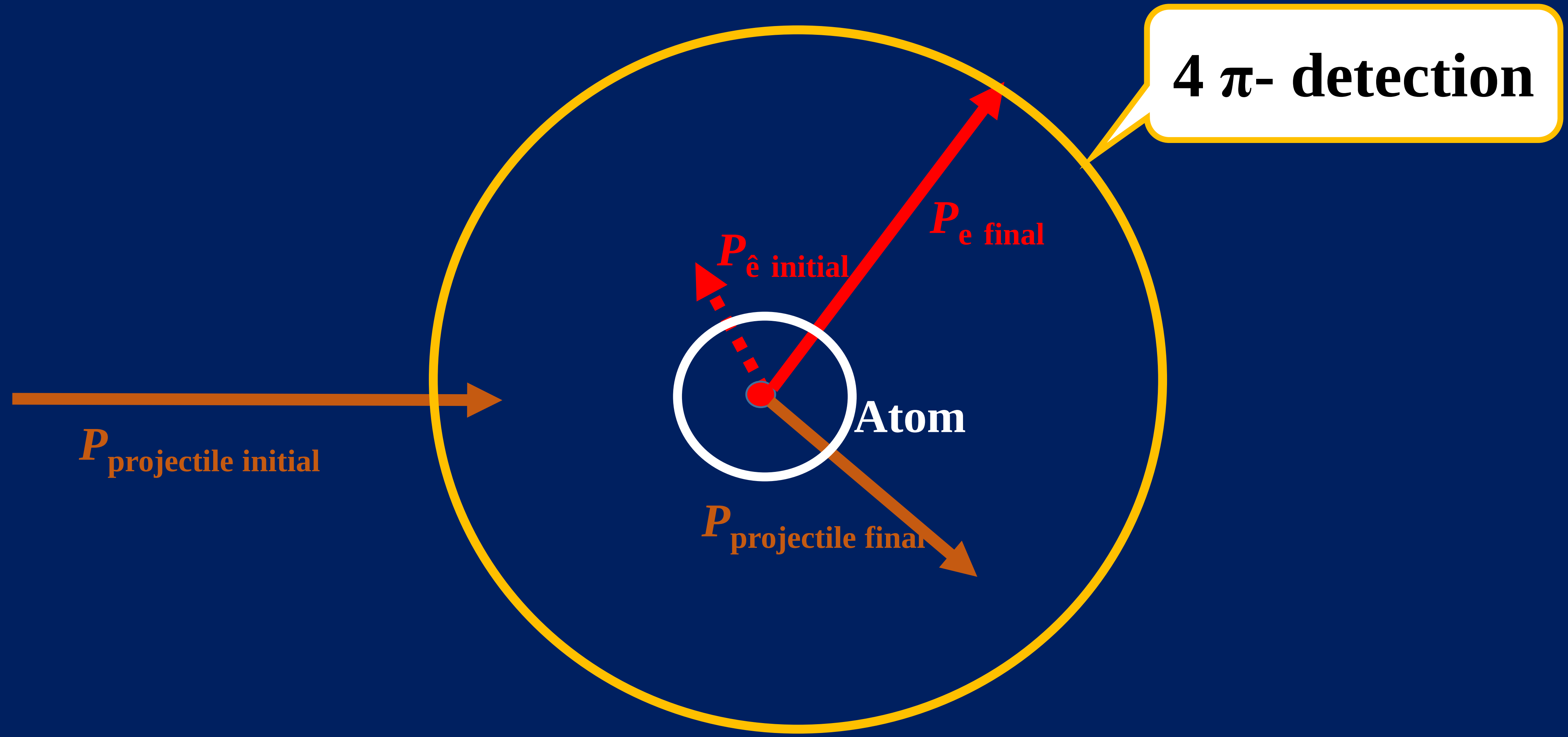
=>

The parameter disturbance in the collision process by the impacting projectile must be measurable or should be negligible.



In the macro world perturbation (e.g. by photons) is so tiny that it not measureable.

Scheme of a single-event coincidence measurement:
e.g. by scattering of a projectile (photon, electron, ion etc.)
on quantum object (atom, molecule, solid state device etc.)
With 4π-detection of the emitted radiation (photon, electron, ion etc.).



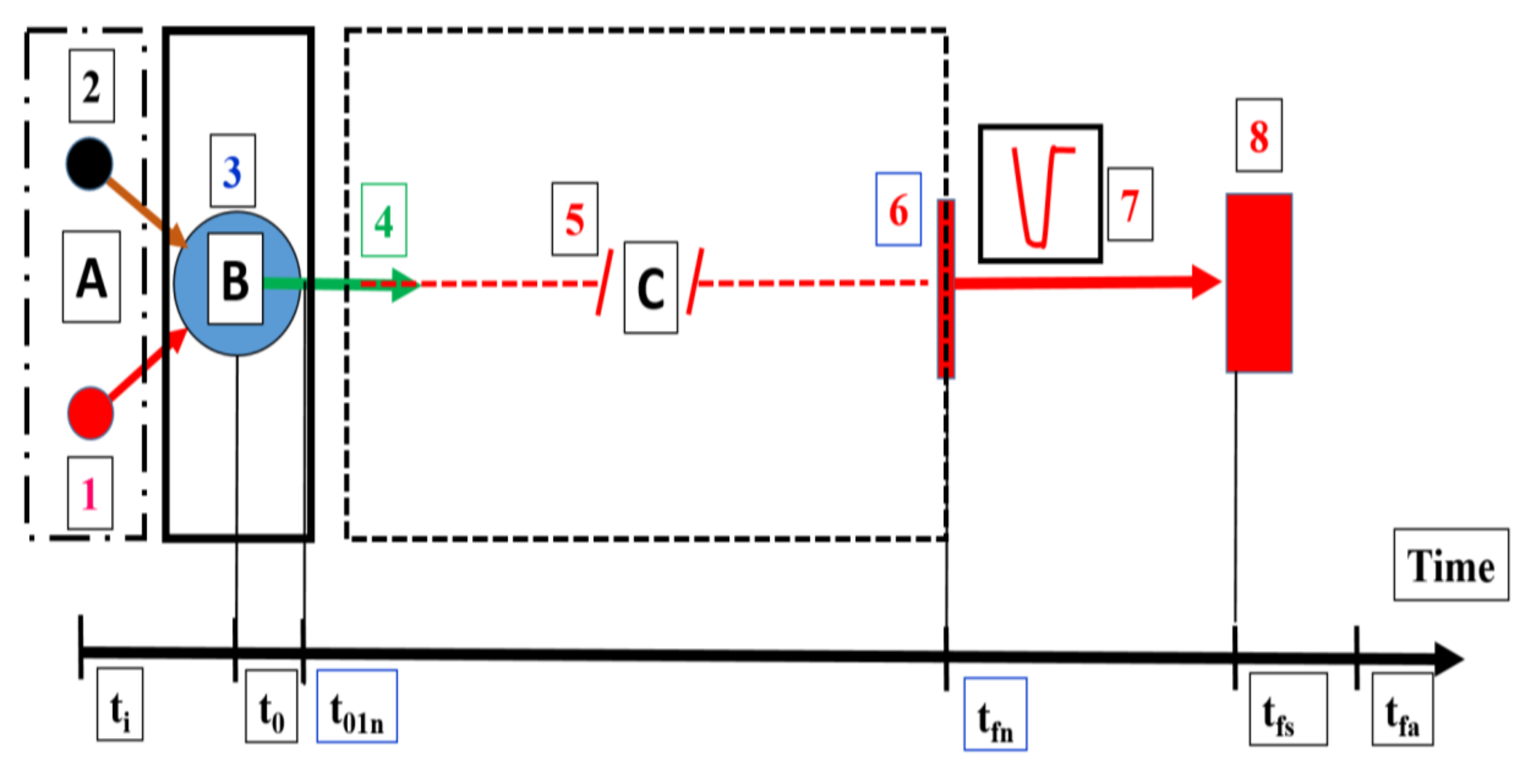
Important: Each measurement takes some time

Time steps of the measurement:

A. Preparation,

B. Reaction (scattering or excitation),

C. Detection of emitted radiation and fragments



To obtain optimal resolution, the measured parameter must obey time-conservation properties!

Werner Heisenbergs scheme of a quantum measurement

Heisenbergs famous paper of 1927

"Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik"

(On the Intuitive Content of Quantum Theoretical Kinematics and Mechanics)

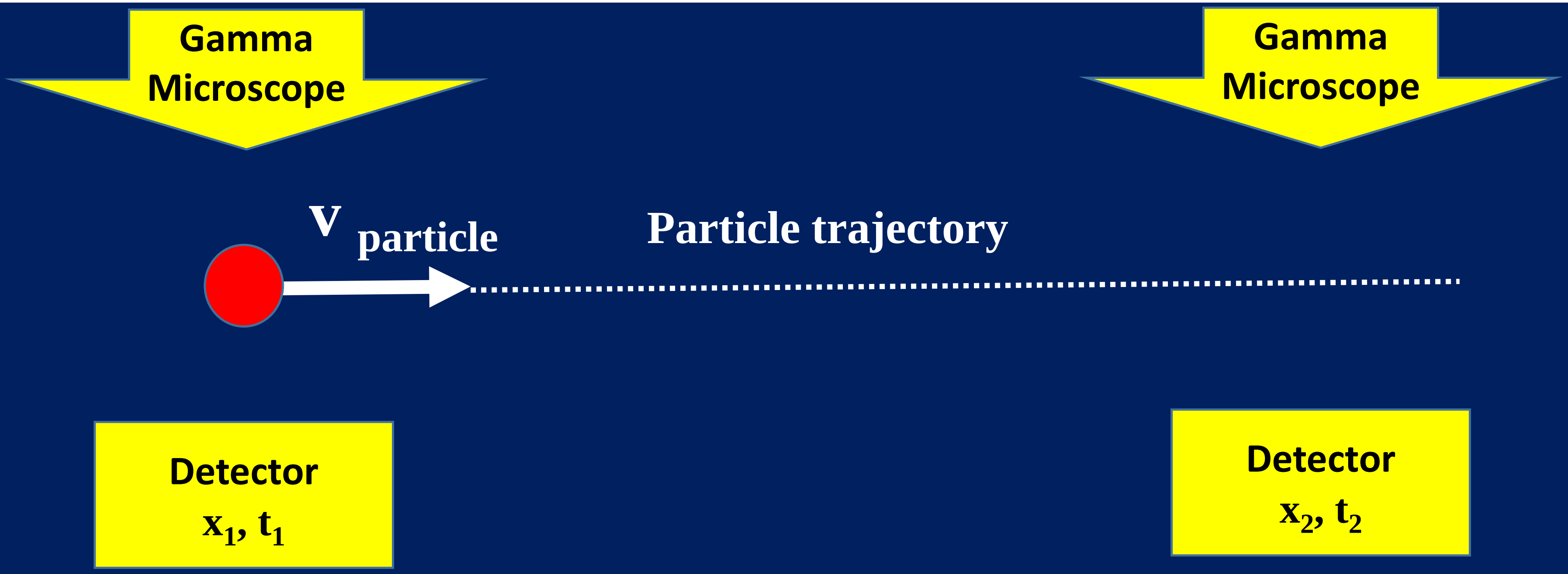
is still for most physicists the "bible" for the measurement accuracy in the quantum world.

Fortunately, it does not apply to a modern state-of-the-art quantum measurement!

To understand this statement:

One must know Heisenbergs concept of a position and velocity measurement in quantum systems?

Heisenberg began in the introduction of his paper with the discussion of a quantum measurement of the position and velocity of a single electron at a given time.



The concept Heisenberg‘s measurement method

Aber daß eine Revision der kinematischen und mechanischen Begriffe notwendig ist, scheint aus den Grundgleichungen der Quantenmechanik unmittelbar zu folgen. Wenn eine bestimmte Masse m gegeben ist, hat es in unserer gewohnten Anschauung einen einfach verständlichen Sinn, vom Ort und der Geschwindigkeit des Schwerpunkts dieser Masse m zu sprechen. In der Quantenmechanik aber soll eine Relation $pq - qp = \frac{h}{2\pi i}$ zwischen Masse, Ort und Geschwindigkeit bestehen. Wir haben also guten Grund, gegen die kritiklose Anwendung jener Worte „Ort" und „Geschwindigkeit" Verdacht zu schöpfen. Wenn man zugibt, daß für Vorgänge in sehr kleinen Räumen und Zeiten Diskontinuitäten irgendwie typisch sind, so ist ein Versagen eben der Begriffe „Ort" und „Geschwindigkeit" sogar unmittelbar

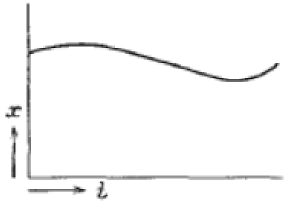


Fig. 1.

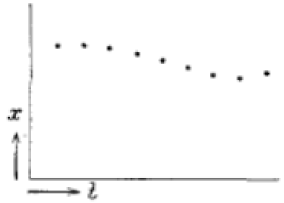


Fig. 2.

plausibel: Denkt man z. B. an die eindimensionale Bewegung eines Massenpunktes, so wird man in einer Kontinuumstheorie eine Bahnkurve $x(t)$ für die Bahn des Teilchens (genauer: dessen Schwerpunktes) zeichnen können (Fig. 1), die Tangente gibt jeweils die Geschwindigkeit. In einer Diskontinuumstheorie dagegen wird etwa an Stelle dieser Kurve eine Reihe von Punkten endlichen Abstandes treten (Fig. 2). In diesem Falle ist es offenbar sinnlos, von der Geschwindigkeit an einem bestimmten Orte zu sprechen, weil ja die Geschwindigkeit erst durch zwei Orte definiert werden kann und weil folglich umgekehrt zu jedem Punkt je zwei verschiedene Geschwindigkeiten gehören.

But the necessity of a revision of kinematic and mechanical concepts seems to follow directly from the fundamental equations of quantum mechanics. Given a certain mass m, it makes simple, understandable sense, in our usual view, to speak of the position and velocity of the center of mass of this mass. In quantum mechanics, however, a relationship $pq - qp = h/2\pi i$ is supposed to exist between mass, position, and velocity. We have good reason to be suspicious of the uncritical application of the words "position" and "velocity." If one admits that discontinuities are somehow typical for processes in very small spaces and times, then a failure of the very concepts of position and velocity is even immediately plausible:

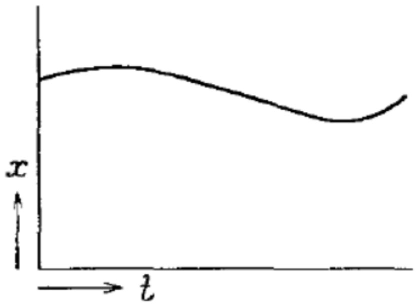


Fig. 1.

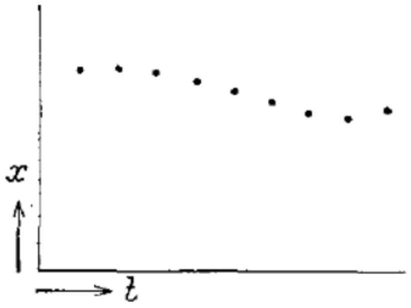


Fig. 2.

If one considers, for example, the one-dimensional motion of a point mass, in a continuum theory one can draw a trajectory curve x(t) for the trajectory of the particle (more precisely, the center of mass) (Fig. 1), the tangent giving the velocity. In a discontinuity theory, however, this curve will be replaced by a series of points of finite distance (Fig. 2). In this case, it is obviously meaningless to speak of the velocity at a specific location, because the velocity can only be defined by two locations and, conversely, because each point corresponds to two different velocities.

But Heisenbergs γ-microscope would never work and remains science fiction!

Why?

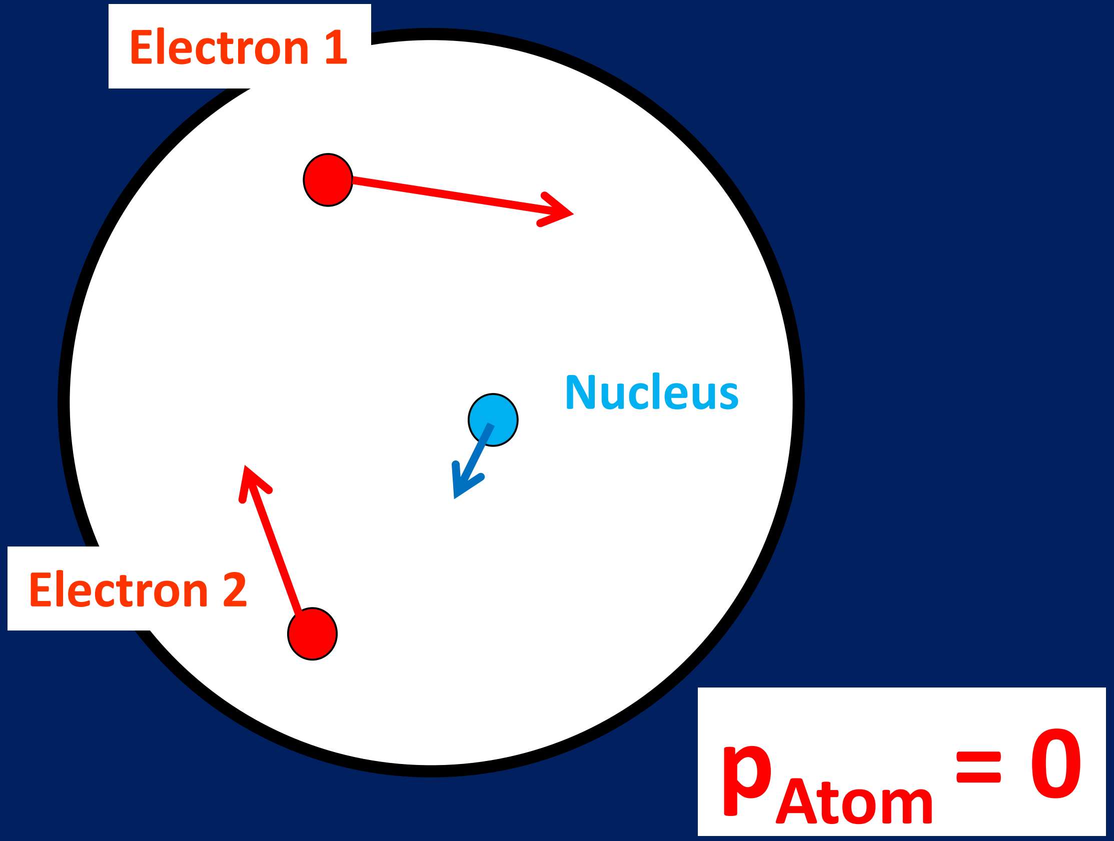
1. One would need a sub-attosec γ- pulse of unrealistic high intensity (energy).

2. The momenta and positions of all electron in the γ- pulse focus would be dramatically changed in such an interaction.

3. One would never know on which electron the γ- pulse is scattered.
Etc.

But how can an experimenter measure intra atomic quantum parameters,
e.g. the true velocity of a bound electron at a specific time?

Measurement of the intra-atomic dynamics in a He atom by multi-coincidence detection e.g. the momentum vectors (velocities)

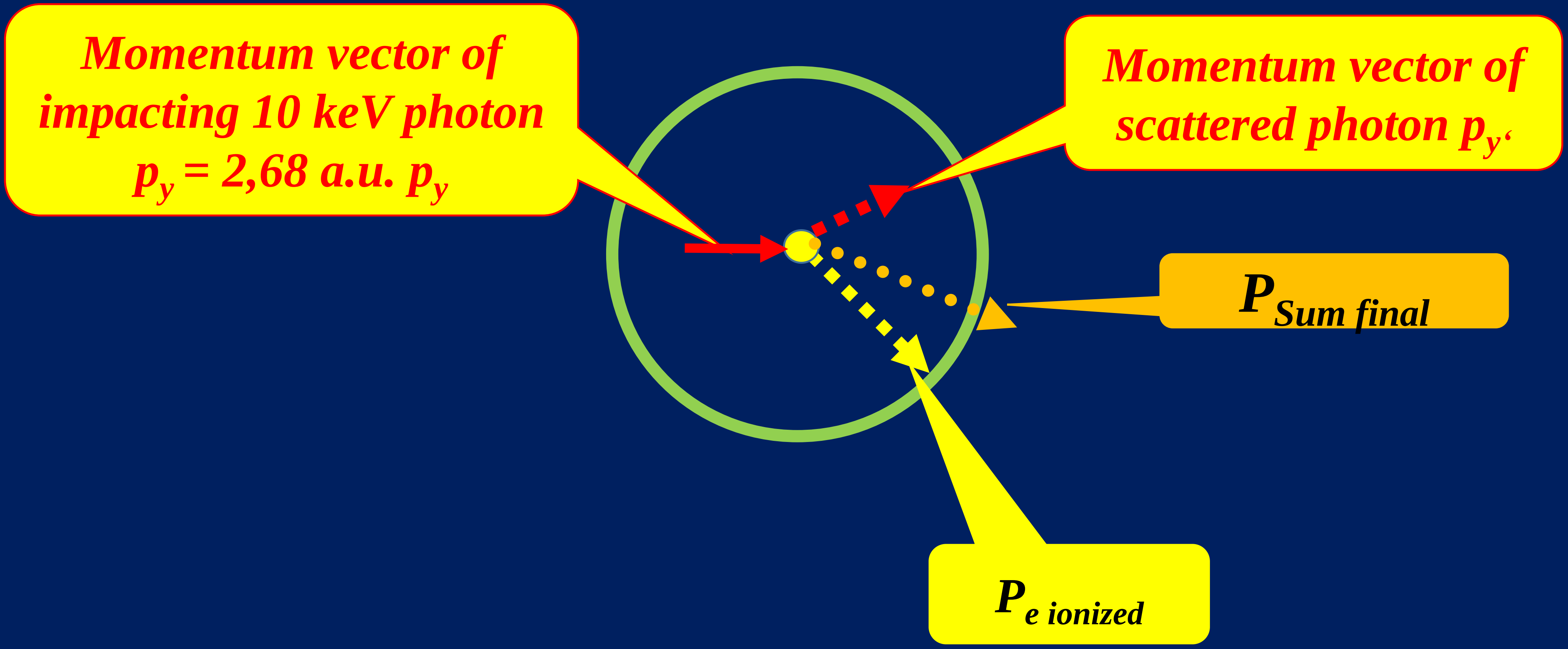


Measure the momenta of all emitted fragments in a Compton scattering process by multi-coincidence technique.

Compton scattering

$$\gamma + \mathrm{He}\,(e_{bound}) \Rightarrow e_{ionized} + \gamma` + \text{Recoil-Ion}$$

The He Atom and Recoil ion momenta are measured too, but they are anyway rather small.



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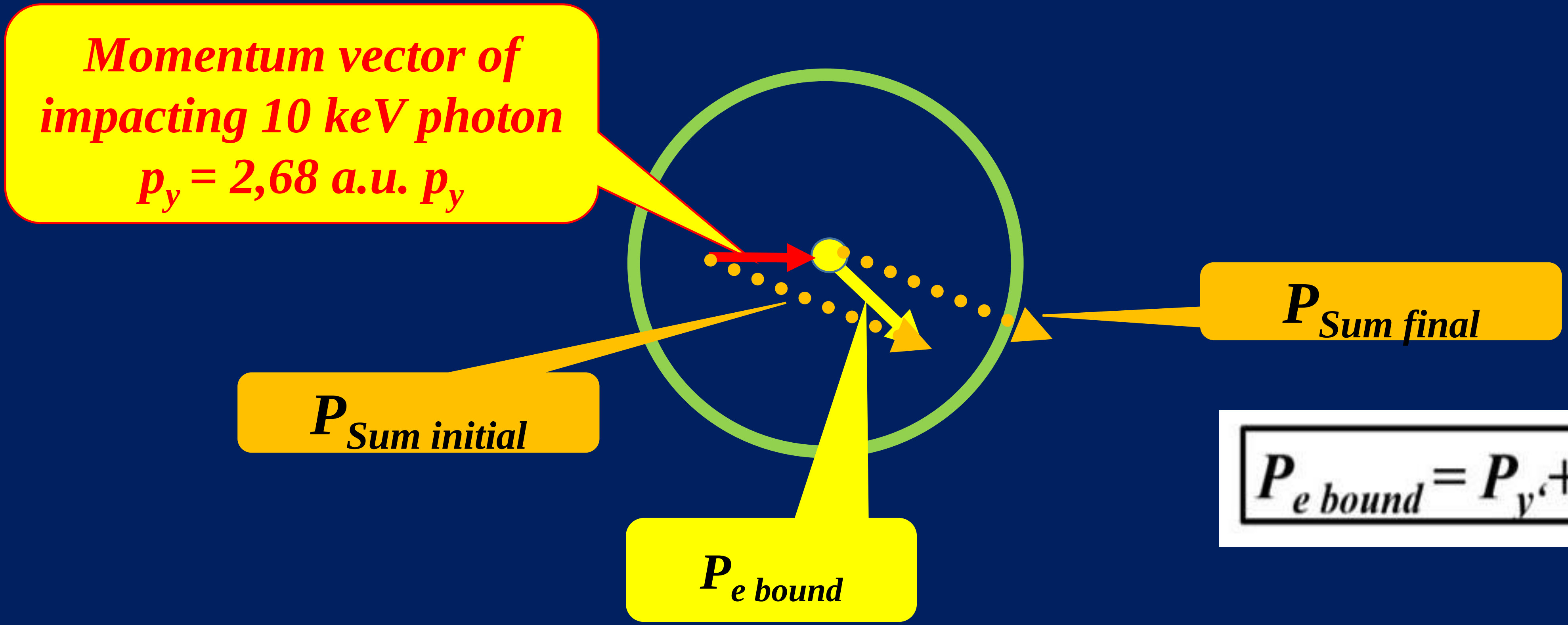

$$P_{e\ bound} = P_{y'} + P_{e\ ionized} - P_y$$

First coincidence experiment was performed in 1924-1926 by Walther Bothe and Hans Geiger

Precondition for an excellent momentum resolution

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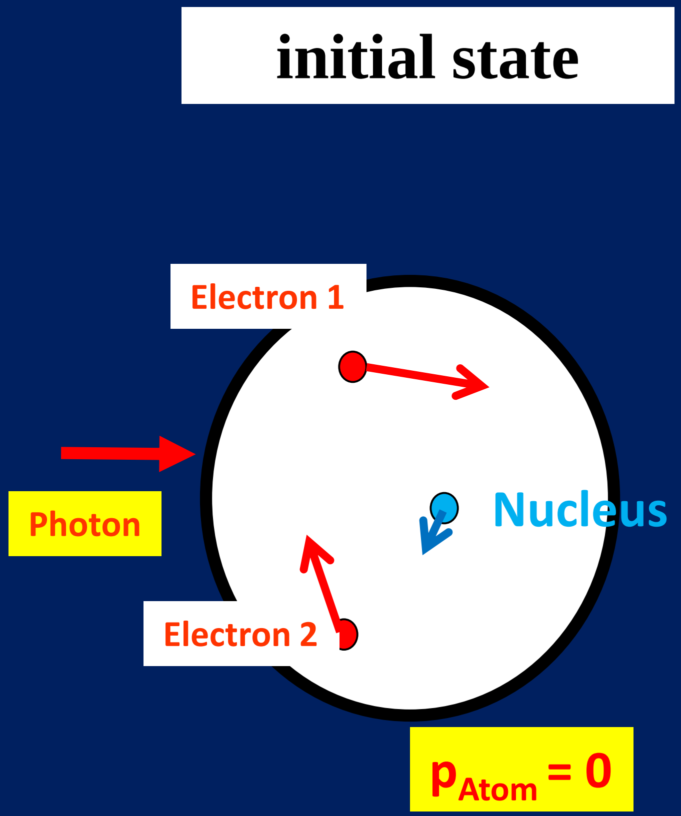
One has to measure in coincidence with high resolution the momenta of all reaction partners in the final- but also initial-state.



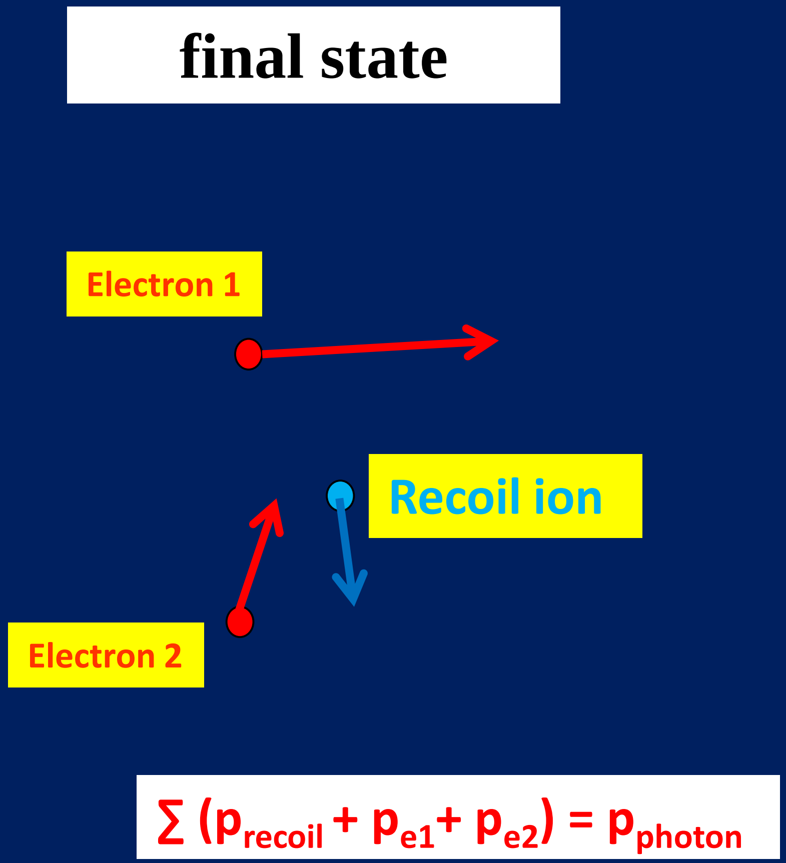
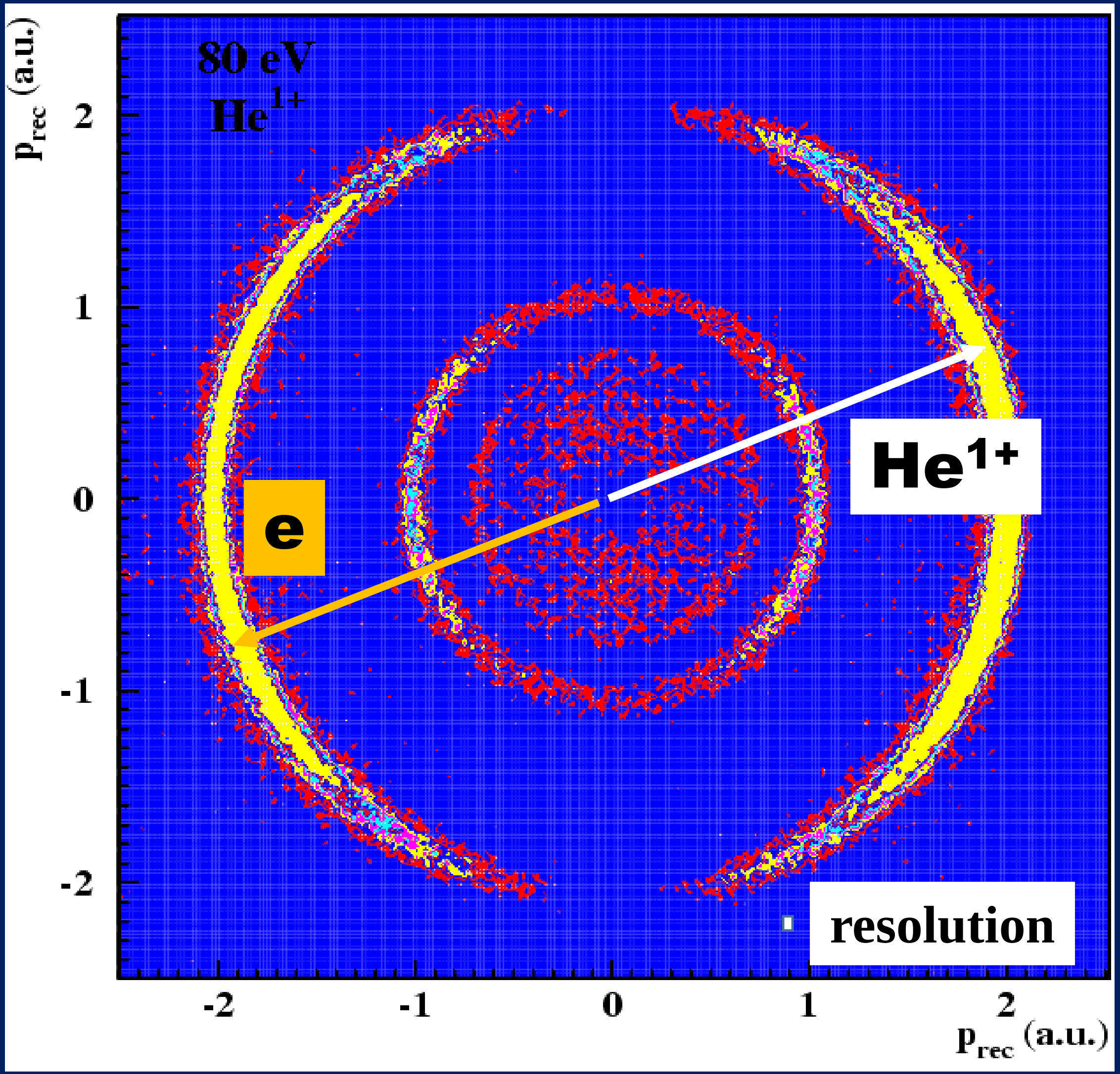
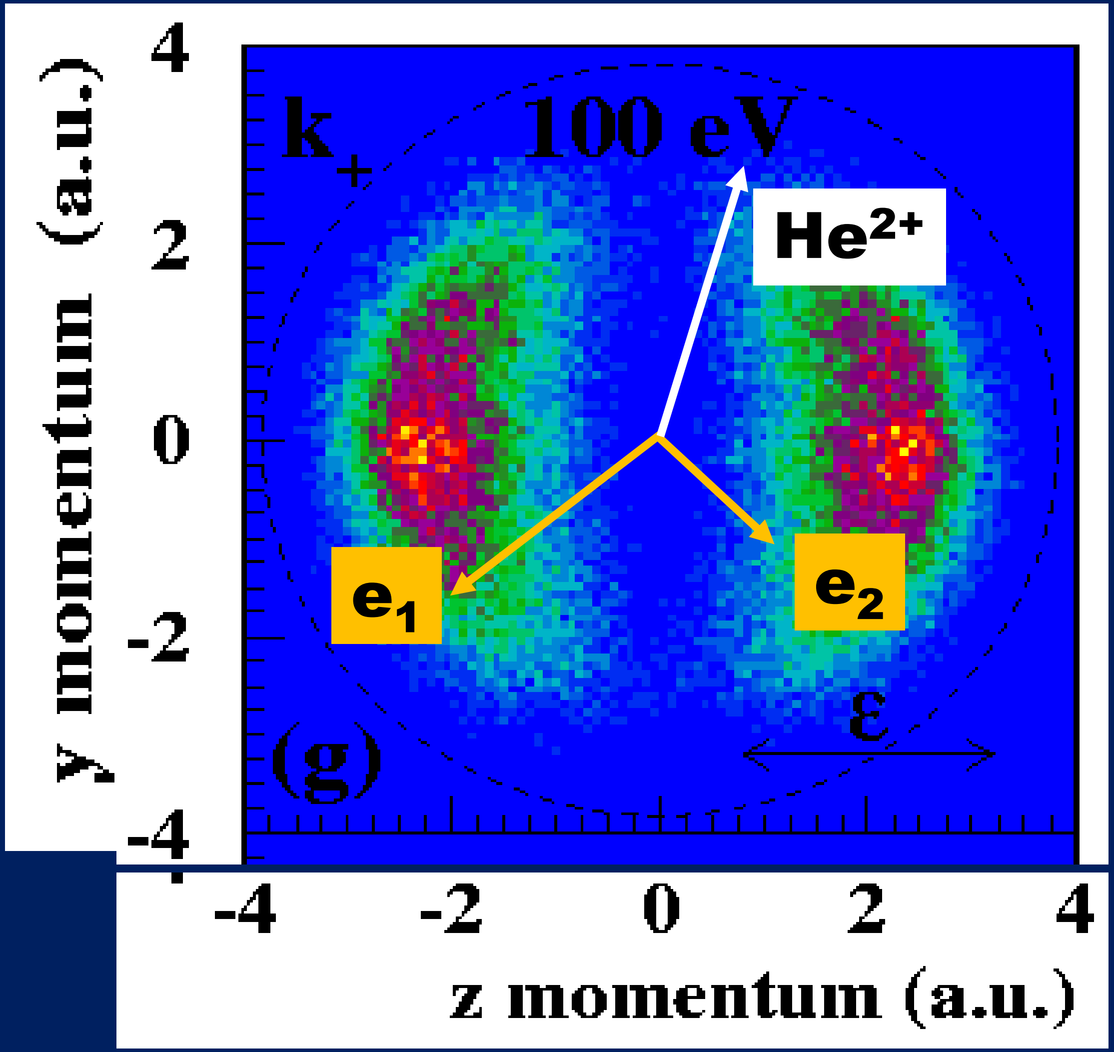


Photo ionization measurements

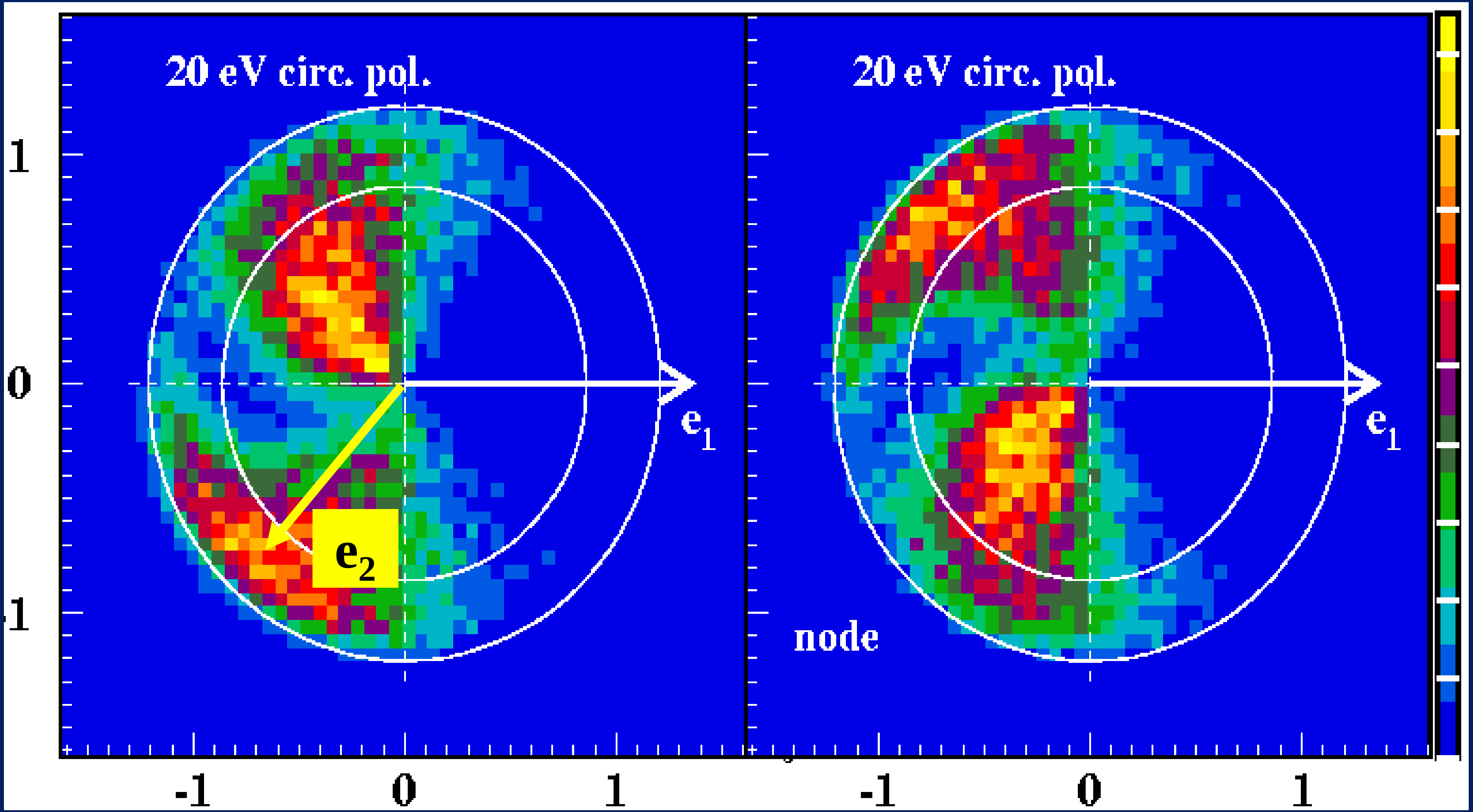


80 eV $\gamma + He \Rightarrow e + He^{1+}$



100 eV $\gamma + He \Rightarrow e_1 + e_2 + He^{2+}$

$\gamma^{circ} + He \Rightarrow He^{2+} + e_1 + e_2$

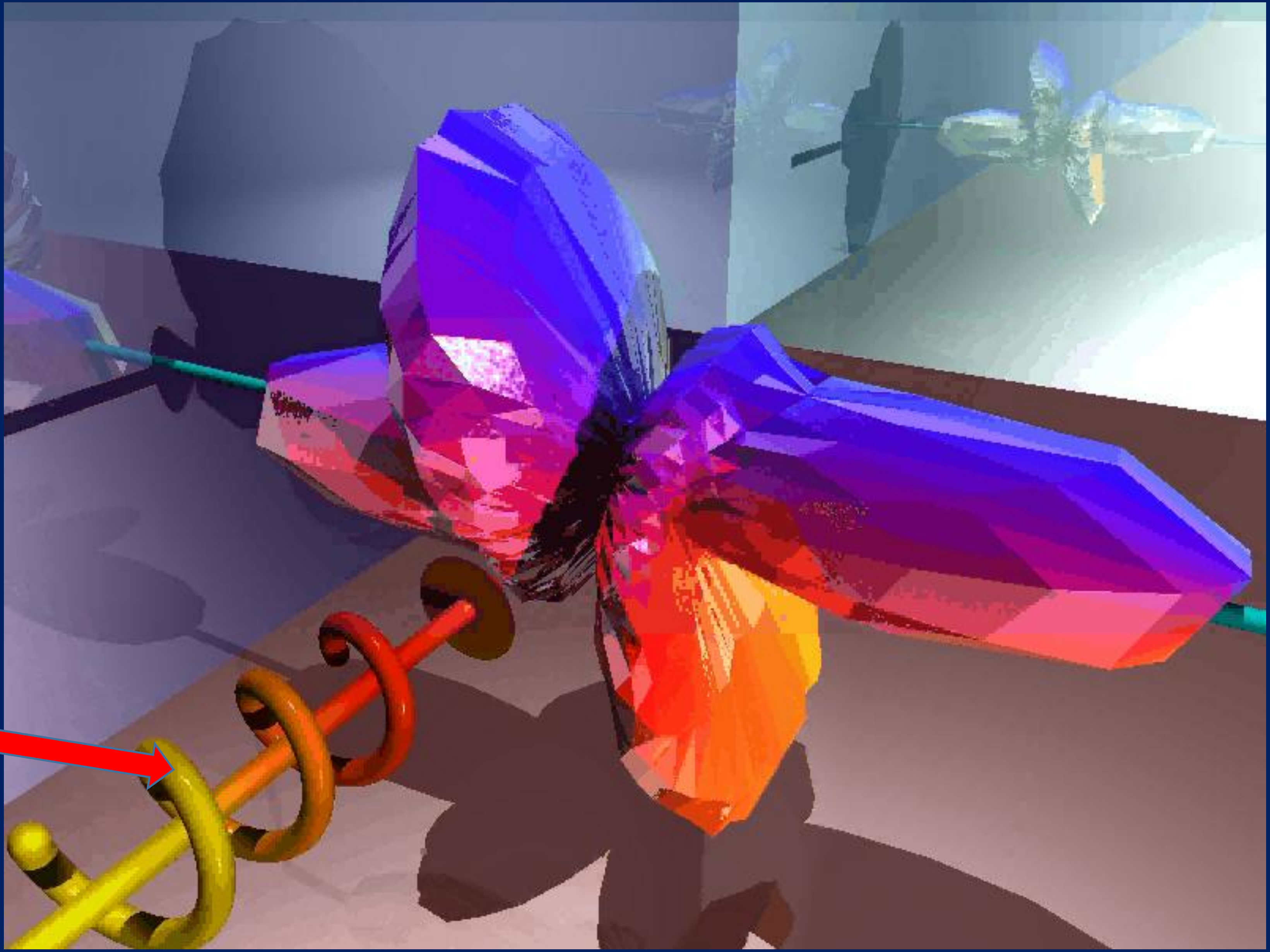


$\gamma^{circ} + N \Rightarrow N^{1+} + e_1$

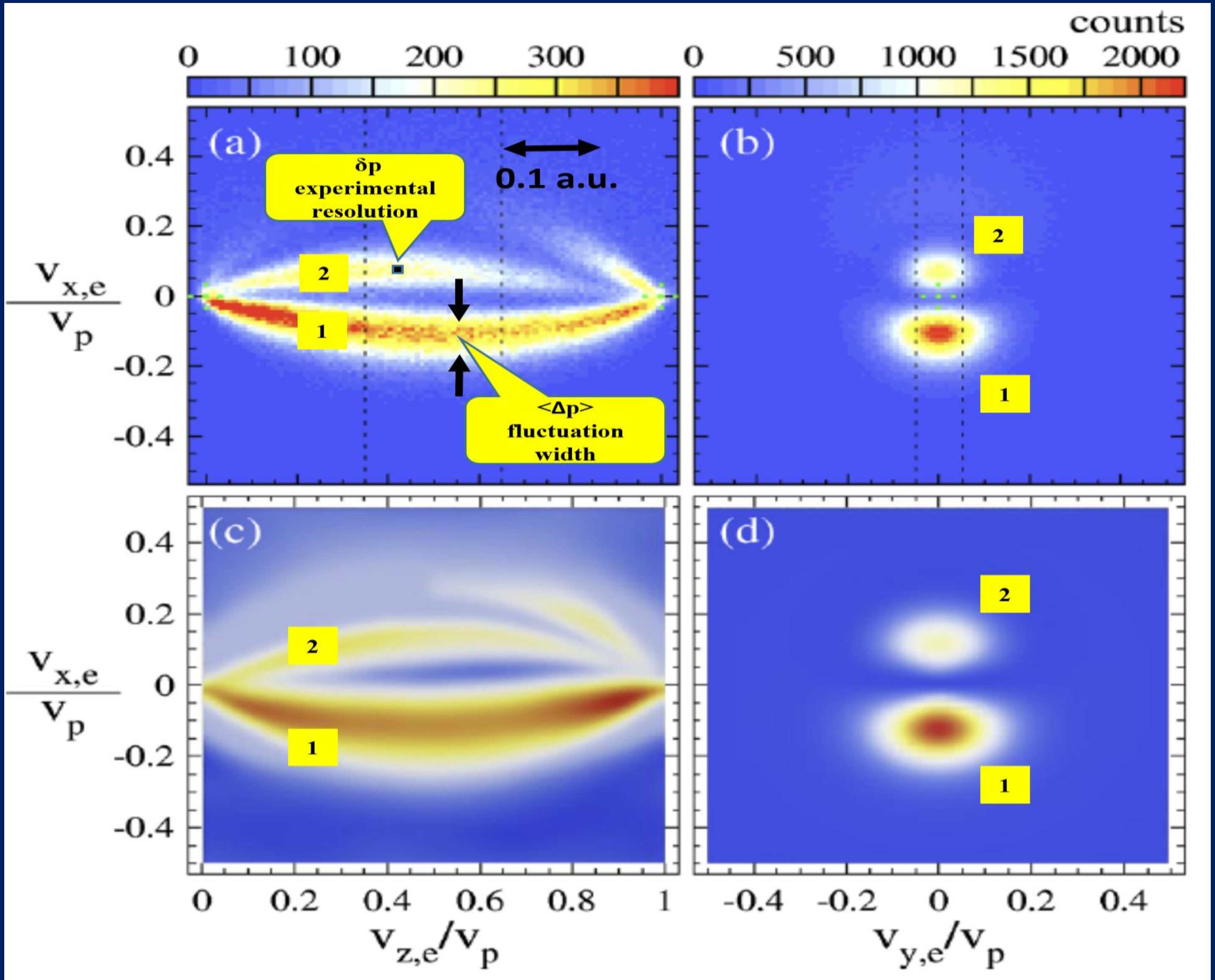
The emission probability of the departing electron as function of the emission angle is represented by the shape of the multipole pattern.

i.e. this is the distribution of electron momentum vectors.

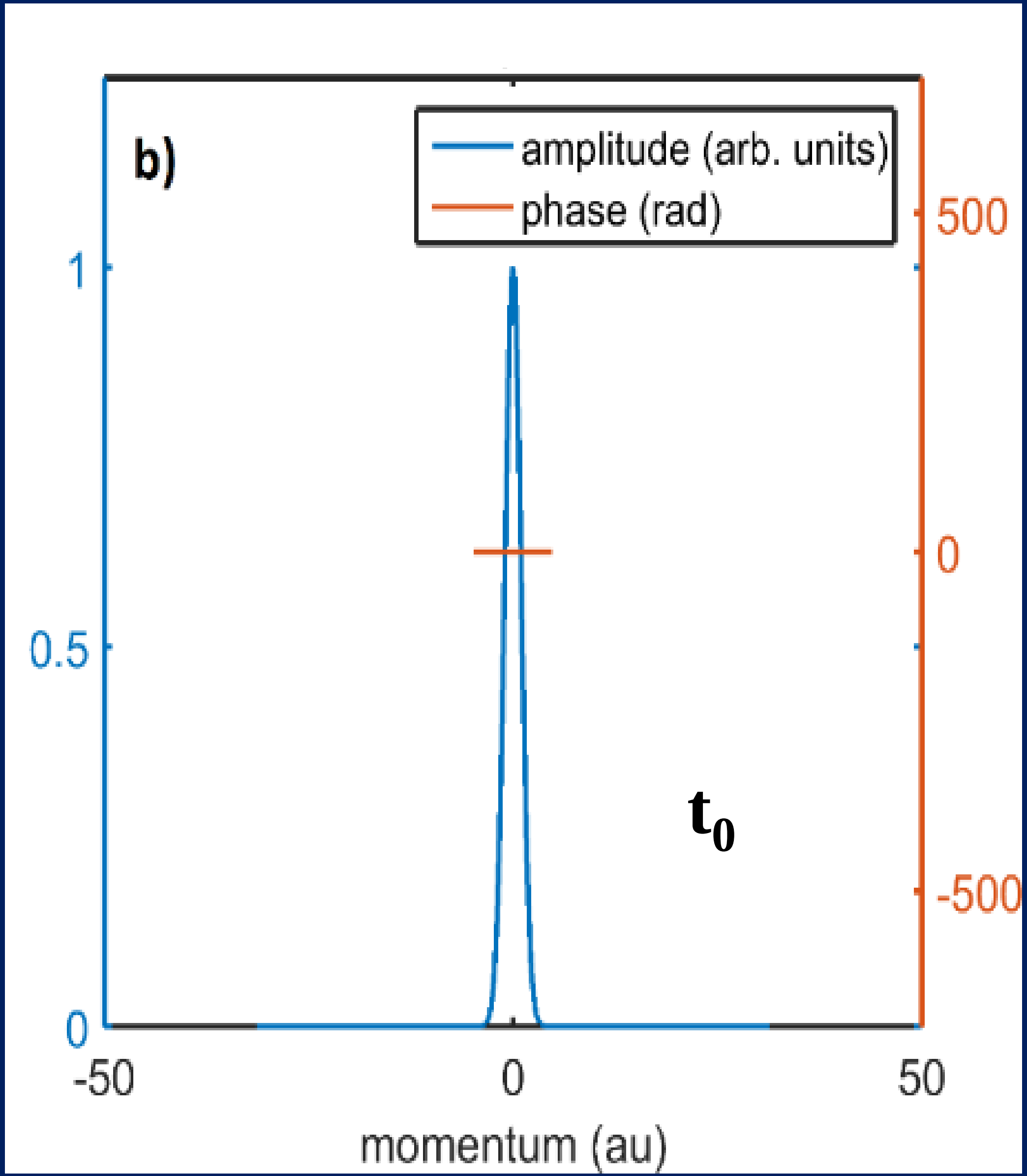
The direction of incidence of the left-circularly polarized photon is represented by the vector.

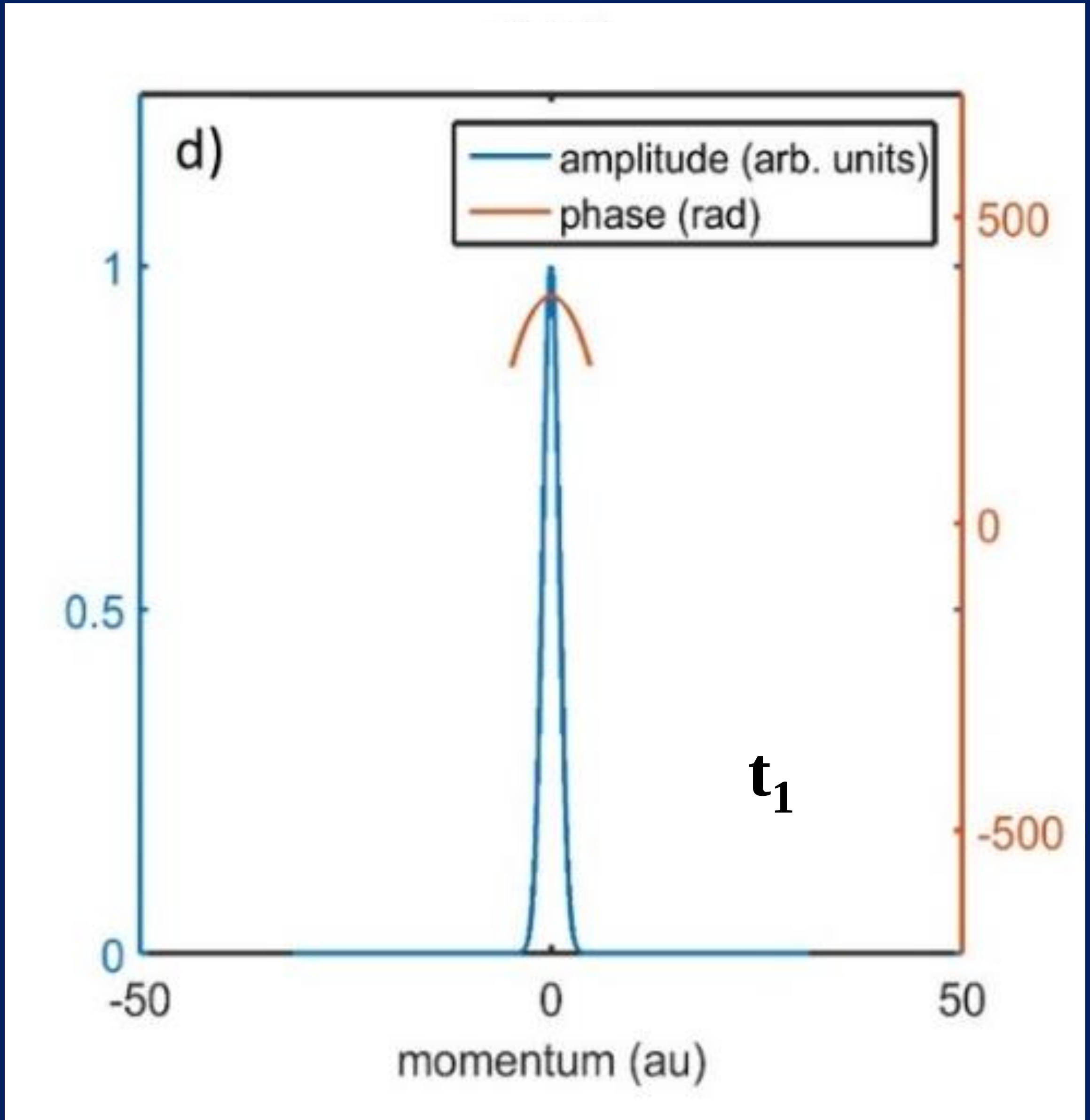


Ion-atom collsion – Transfer ionization channel
10 keV He^{2+} + He => He^{1+} + He^{2+} + e



Theory: Time evolution of the momentum wave function

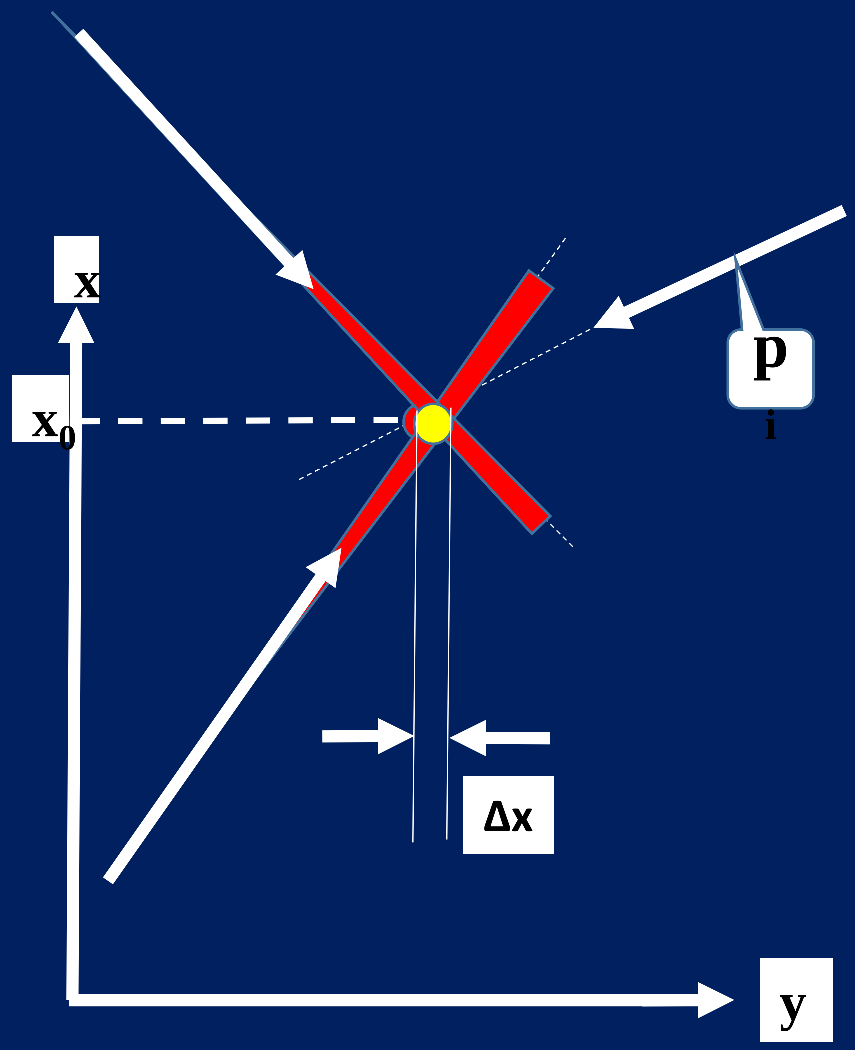




Determination of position in a single-event measurement

In a multi-coincidence single-event measurement
the position x_0 could be determined
as intersection point (yellow dot)
of at least two or more measured momentum vectors.

Achievable resolution is Δx.



The detector impact position is the zero position of the momentum vector
It can be measured with an accuracy of Δy (typically 0,1mm but 10 a.u. also possible).

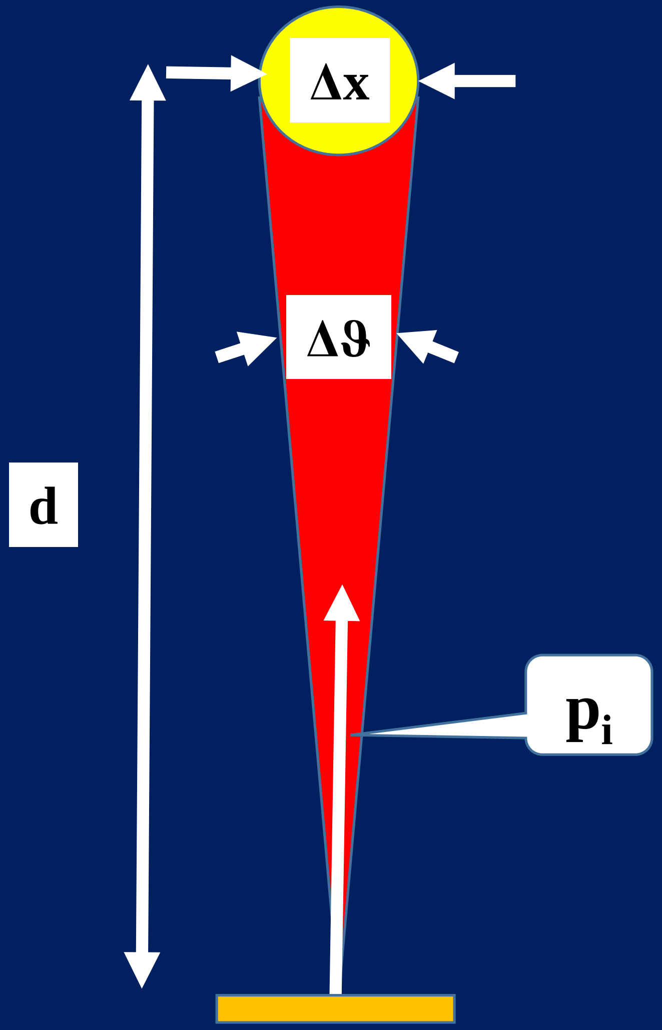
Since the momentum vector has an angular spread of $\Delta\vartheta$ (red cone)

the cone has in distance d from the detector a width of Δx

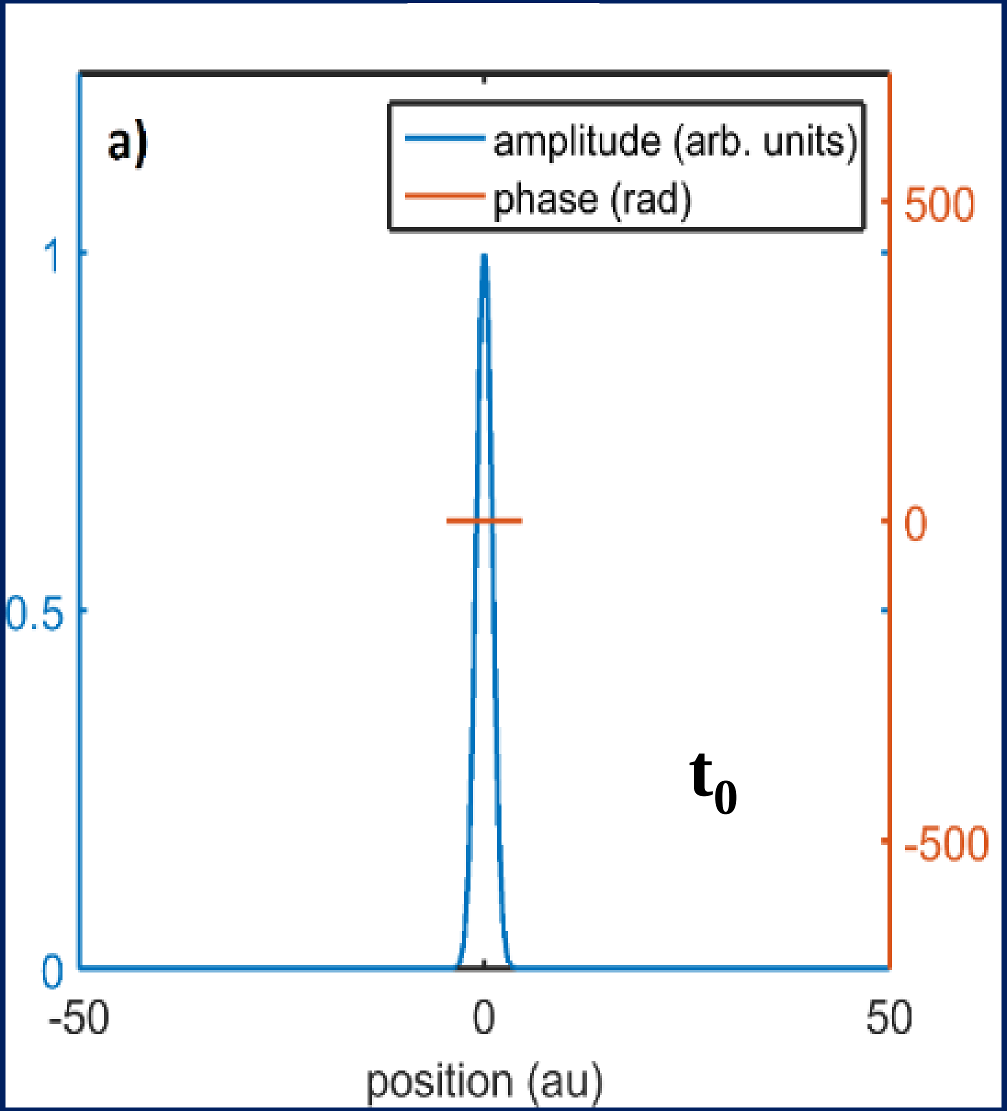
The achievable position resolution Δx is thus
$\Delta x = \Delta\vartheta \cdot d = (\Delta p_i / p_i) \cdot d.$

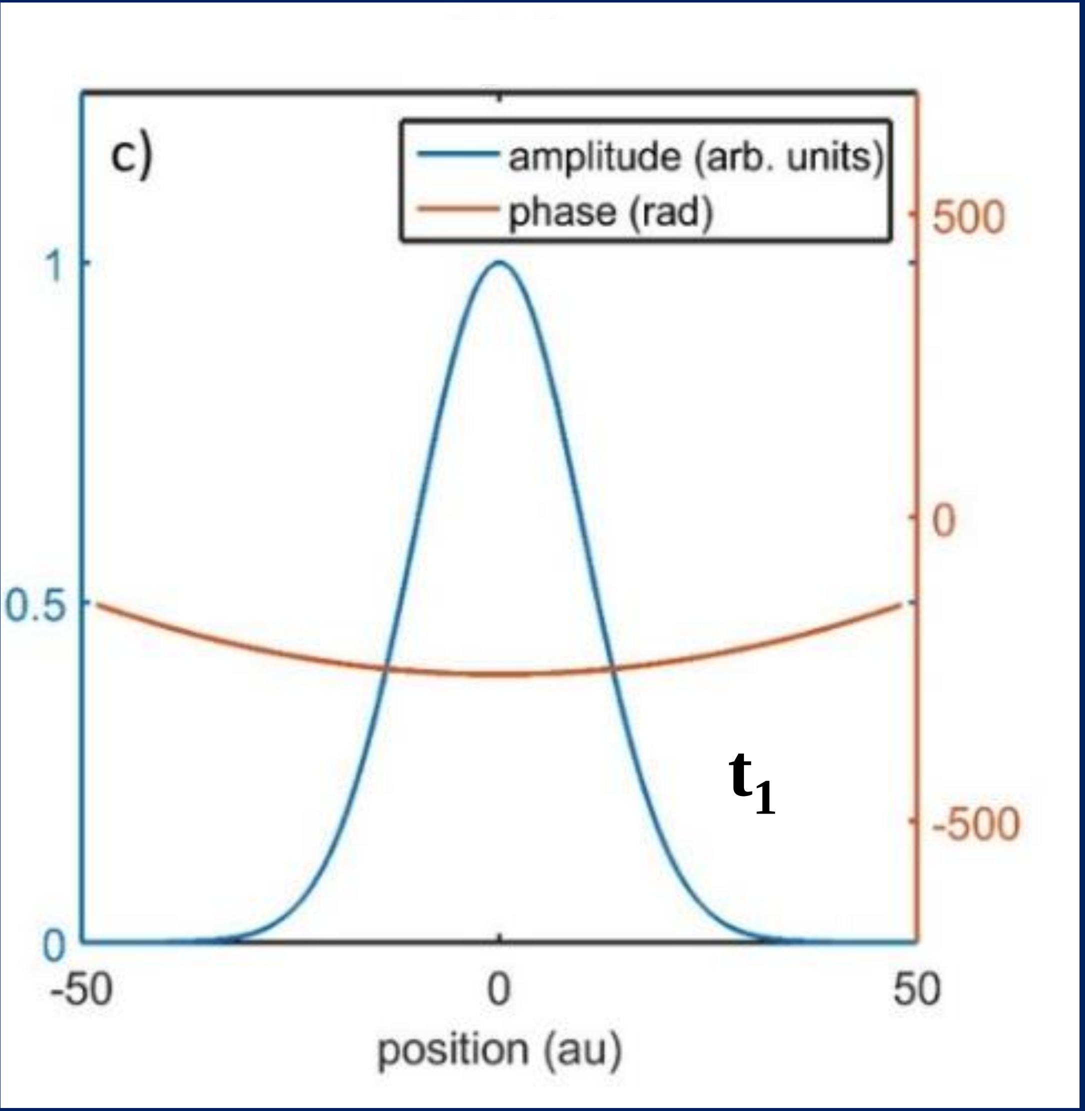
Although
$\Delta\vartheta = (\Delta p_i / p_i)$ can be made even smaller than 10^{-4} by increasing d.

But since d has macroscopic dimensions of more than 10^{+9} a.u. ,
Δx gets worse with increasing d and is in the order of 10^{+5} a.u.. = 0,01 mm.



Time evolution of the position wave function





How small can the product of $\Delta p \bullet \Delta x$ be made in a single event measurement?

In case the position sensitive detector provides a resolution $\Delta x \approx 10$ a.u.
and the momentum resolution is $(\Delta p_i / p_i) \approx 10^{-3}$
=>
Then the products $\Delta p_x \bullet \Delta x$ and $\Delta p_y \bullet \Delta y$ in the moment of impact on the detector
can be made in x- and y-direction (perpendicular to vector p)
smaller than 10^{-2} a.u.,

but in z-direction (flight direction) it remains >>> than 1 a.u.

Conclusion:

We have shown that in a single reaction between quantum particles at a given time, only the momenta of the emitted particles can be measured with not limited subatomic resolution, but not their positions. This fundamental discrepancy between the conjugate variables momentum and position is due to the fact that the momentum of a free particle is conserved as a function of time but position is not!

The achievable resolution in momentum space Δp in a single event process, when using "Time-of-Flight" TOF methods, improves linearly with TOF t and has in principle no lower limit, where t is the particle flight time between reaction moment and impact on the detector. Whereas the position resolution linearly decreases with TOF t.

We emphasize that (contrary to popular belief) Heisenberg's "uncertainty principle" UR does not limit the achievable resolution of the momentum in a single-event measurement. Heisenberg's statement that in a single-event measurement only either the position or the momentum (the velocity) of a quantum particle can be measured with high precision is false and thus contradicts theory and a real experiment.

The UR rule simply indicates only a "correlation" between the mean statistical fluctuations of a large number of repeated "dynamically not-correlated" single-event measurements of two conjugate variables.

Questions and Thoughts of an experimental physicist:

what prevents the electron shell, according to the prediction of classical physics within some femtosec to auto-ionize and to create a plasma state?

To avoid the autoionization the shell must form a dynamically entangled UNIT.
What kind of glue hold the shell together?

Analogy: if in cars or in air planes all pieces would not be screwed or glued together, when moving the car or plane they would immediately fall into pieces.

It cannot be the Coulomb force alone,
angular momentum and its quantization with 4 degrees of freedom (4 quantum numbers) must provide this glue and create a dynamically entangled UNIT.

This entangled dynamics must be able too, to store all the information which enables bio-systems to be alive and to create growth

As seen in the lecture by Reinhard Dörner
With the COLTRIMS-Reaction Microscope one can resolve this dynamics below the 10 Zepto-sec region.

Each measurement looks backward in time and determines the parameters of the past and not of the future (Karl Popper).

Heisenbergs „Uncertainty Relation“ predicts the future.